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Guidelines for incorporation of vehicle–pavement interaction effects in pavement design

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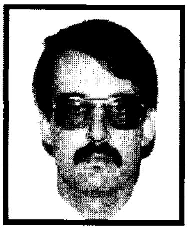
Vehicle–pavement interaction defines the relationship between a vehicle on a pavement, and the pavement. Pavement unevenness causes vehicles to generate dynamic tyre loads. The pavement roughness level, vehicle type, vehicle components, and vehicle speed affect these tyre loads. Application of these loads to the pavement cause transient pavement responses that differ from static pavement response to static loads. Incorporation of these dynamic tyre loads into pavement design is a complicated mathematical process. A guideline based on existing pavement analysis procedures was developed to simplify this process. In this paper a short background to vehicle–pavement interaction is provided, followed by the guideline and an example of applying the guideline. The objective of the paper is to expose the pavement engineer to a practical guideline for incorporation of vehicle–pavement interaction issues into pavement design. Application of the guidelines indicated that use of dynamic tyre load populations rather than equivalent static loads in pavement response analyses can cause significantly different pavement responses.



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INTRODUCTION

Pavement structures consist of various natural and engineered materials that react to load input from vehicles in specific ways. As tyre loading is a time-dependent dynamic parameter, the response of the pavement system to the input is time-dependent and transient. Owing to a lack of computational power, models and data, pavements have traditionally been analysed in a static mode, where it is assumed that both the load input and the pavement response are static and time-independent. Pavement engineers have long realised that this is not the actual situation, and various efforts into incorporating dynamic loading and transient pavement response were made.

Although different methods exist for incorporating moving dynamic tyre loads and transient pavement response into pavement design, there are limitations regarding issues such as tyre load characterisation, material models, the ability to model non-symmetrical load conditions and the material properties to be used in the models. A need exists to provide a practical and user-friendly approach to transient pavement response analysis to real moving tyre loads.

In a study performed at CSIR Transportek a guideline for evaluating vehicle–pavement interaction from a pavement design viewpoint was developed. The study looked at issues such as a holistic framework for vehicle–pavement interaction, vehicle and pavement components, vehicle–pavement interaction parameters and the development of practical guidelines for incorporation of vehicle–pavement interaction into pavement design. In a previous paper certain aspects of the study have been discussed (Steyn & Visser 2000).

In this paper a short background to the overall project is provided. The origin of some of the data and equations used in the guidelines is discussed. It is followed by the guidelines and an example of the incorporation of moving dynamic tyre loads into pavement design and analysis. Finally, some unresolved issues in the field of vehicle–pavement interaction are discussed. The objective of the paper is to expose the pavement engineer to a user-friendly and practical guideline for incorporation of vehicle–pavement interaction issues into pavement design and analyses.

BACKGROUND

Generically tyre loads vary in two ways. These are the variation of load between vehicles travelling on a pavement, and the varying loads applied by a vehicle along the pavement. The first type of variation is accommodated in pavement analysisthrough equivalent load concepts. The second type of variation is caused by the pavement roughness-induced movement of the vehicle. This is traditionally termed dynamic pavement loading (DADS 1997). The use of the term dynamic loading in pavement analysis refers to a load with a constant magnitude that is moving along a pavement or a load that is varying in load magnitude. The following four definitions should clarify the terminology.

The load magnitude of a Static Load is independent of time and position, and the position where it is applied is independent of time. For a Moving Constant Load the load magnitude is constant but the position where it is applied is dependent on time. For a Dynamic Load the load magnitude is dependent on time but not on position, and the position where it is applied is independent of time. The load magnitude of a Moving Dynamic Load is dependent on both time and position and the position where it is applied is also dependent on time. Real traffic cause either Static Loads or Moving

Dynamic Loads, while Dynamic Loads and Moving Static Loads are mainly used in research to simplify the understanding of pavement response. Two types of pavement response analysis are defined. Static Response Analysis indicates that pavement response parameters are independent of time but dependent of load position, while Transient Response Analysis indicates that pavement response parameters are dependent of time and of load position.

HEAVY VEHICLE TYRE LOADING

Detailed tyre load functions were needed to evaluate the response of pavements to load conditions. The input data for this project originated from a fingerprinting of South African heavy vehicles (Steyn & Fisher 1998), while tyre load histories were simulated using the Dynamic Analysis and Design System (DADS 1997) software. Three typical vehicles were simulated as running over three pavements having different pavement roughnesses. These were a rigid vehicle with two axles, an articulated vehicle with six axles and an interlink combination with seven axles. Three typical speeds and load conditions were used for each vehicle (Gilliomee 1999).

The statistical tyre load data analysis (Steyn 2001) indicated that the average heavy vehicle loads were affected statistically significantly by the Gross Vehicle Mass (GVM) per tyre, independent of the speed and pavement roughness (equation 1). The Coefficient of Variation (CoV) of the tyre loads showed good relationships with the vehicle speed, vehicle type, pavement roughness, vehicle load and GVM (equation 2). The statistical analyses also indicated that the tyre loads were normally distributed. This is in agreement with other researchers (ie Sweatman 1983).

Average Load [N] = 12,6
+ 1,003* (Gross vehicle mass [N]
/Number of tyres on vehicle)
R^2 = 99,9%
Standard error of y-estimate = 97,1

Equation 1: Relationships between Gross Vehicle Mass, vehicle type and average tyre load

Coefficient of Variation of Load
= 0,39 - 4E-7*Gross vehicle mass [N]
- 0,003*load [%]
+ 0,01 *number of tyres on vehicle
+ 0,03*roughness [HRI]
+ 0,001*speed [km/h]
R^2 = 94,9%
Standard error of y-estimate = 0,055
Where load indicates the percentage of full load that the vehicle carries
HRI indicates Half-car Roughness Index

Equation 2: Relationship between Coefficient of Variation of tyre loads (CoV Load) and vehicle speed, pavement roughness and vehicle type

The limitations and assumptions used in the development of the equations presented are addressed in the references (Steyn 2001). All the equations and relationships presented are empirical. Equations 1 and 2 incorporate the assumptions of steel suspension, tyre inflation pressures at manufacturers' recommended levels, rigid, articulated and interlink vehicles in the vehicle population, the speed spectrum (40–100 km/h), load spectrum (empty, full and 10% overloaded) and roughness spectrum (Half-car Roughness Index (HRI) = 1,2; 3,1 and 5,3) as used for development of these equations.

The main effect of variations in vehicle speed and pavement roughness is directly proportional to variations in the coefficient of variation of the tyre loads. Increases in any or both of these parameters cause a wider distribution of the tyre loads around the mean, resulting in a higher proportion of peak loads applied to the pavement. As the damage caused by tyre loads on a pavement constitutes an exponential relationship, these increased peak loads cause increased damage to the pavement. All of this will happen with the same average GVM on the vehicle population. An increase in GVM will shift the whole distribution, but will have limited effect on the CoV.

As the distribution of heavy vehicle tyre loads is normal, the average and Coefficient of Variation can be calculated (equations 1 and 2) for a given population of vehicles, speeds and pavement roughnesses, and the expected distribution of dynamic tyre loads for this population determined.

PAVEMENT RESPONSE

Transient pavement response defines the response of a pavement to a moving load input. The effects of vehicle speed, pavement mass inertia and damping on pavement response were included in the analysis (Steyn 2001). Only linear elastic material models were used for the response analyses, and existing transfer functions from the South African Mechanistic Design Method (SAMDM) (Theyse *et al* 1996) were used to evaluate the expected lives calculated.

Results of the static pavement response analyses indicated that increased load magnitude resulted, as expected, in increased stresses, strains and deflections. Similar trends were observed from the various pavement response parameters for each of the pavement structures investigated.

The transient pavement response analysis was performed using a two-dimensional axi-symmetric finite element package (Jooste 1999). The tyre loads used for the transient analyses originated from the tyre load distribution obtained from evaluating all the moving tyre loads of all three vehicles. Tyre loads at the 50th, 80th, 90th and 95th percentile values were used as input data.

The transient pavement response analyses consisted of analysing the response of five pavement structures to moving constant loads. Deflection and strain values generally decreased with increased speeds. Stress values generally remained constant with increasing speeds. Load magnitude showed good relationships with the stresses in the upper part of the pavement structure, while load speed showed good relationships with the strains in the pavement structure. Lower load frequencies affected calculated strains in the deeper parts of the pavement more than higher load frequencies and also affected calculated stresses in the pavement more than calculated strains. The deeper parts of the pavement were less affected under moving constant loads than under static loads. A distance lag existed between the position of maximum load application and the positions of maximum pavement response.

The expected pavement lives calculated using the response parameters from a moving constant load analysis caused higher expected lives than when doing the calculation using static load data.

The analysis of the data from all the pavement response analysis methods at all load and speed conditions indicated that the calculated stresses were not affected by load speed to the same magnitude than the calculated strains.

In order to provide a practical alternative for inclusion of load speed and pavement mass inertia and damping effects in pavement life calculations without the availability of appropriate methods to calculate the pavement responses incorporating these effects or transfer functions for these conditions, an approximate method was developed. This method can be used to estimate the pavement response parameters under moving loads when the static responses are available. The proposed method consists of a set of empirical equations (for different materials and layers) with which the static stresses, strains and deflections as calculated using the linear elastic static software (ie ELSYM5M) can be adjusted for the specific speed at which the results are required. These equations were derived based on a population of the most typical vehicles and pavements currently found in South Africa (Steyn 2001).

The equivalent dynamic response parameters calculated using the empirical equations can be used in the normal SAMDM transfer functions to calculate the expected life of the pavement. The main limitation is that the equations are empirical. However, these equations should provide the pavement engineer with a good estimate of the expected variations in pavement response due to the effect of load speed.

GUIDELINES

TRH4 type guidelines

The guideline for incorporation of vehicle-pavement interaction effects is structured similarly to the TRH4 (1996) document to enable practitioners to understand the changes that need to be included in a pavement design when incorporating speed and vehicle-pavement interaction effects. The process is structured in such a way that the practitioner does not need access to a finite element programme, but that a linear elastic multilayered code can be used together with the currently available SAMDM transfer functions (Theyse *et al* 1996).

TRH4 structure

The standard TRH4 structure for the detailed road pavement design process is shown in figure 1. Only the sections on Pavement design (section 3), Design traffic and pavement class (section 4) and Structural design and pavement type selection (section 8) are affected by the proposed method.

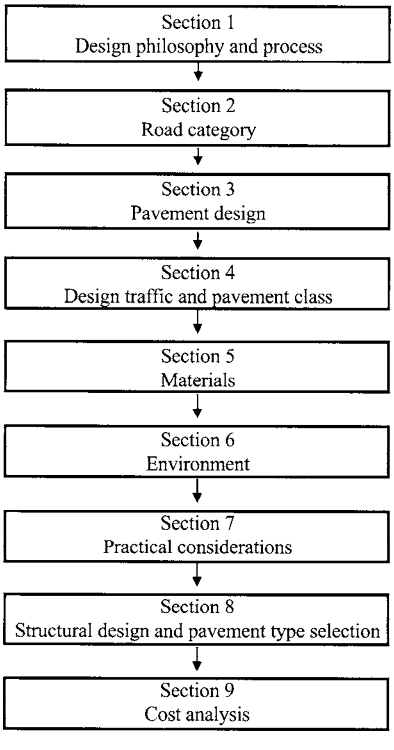

Figure 1 Detailed road pavement design process (TRH4 1996)

Pavement design

Pavement design is concerned with selection of the structural capacity, analysis period, structural period, and pavement structural balance. The required structural capacity is mainly affected by the proposed method. The main difference arises from the method by which the traffic spectrum is calculated for the pavement to be analysed. Currently the traffic spectrum consists of static tyre loads. The new approach entails development of a moving dynamic tyre load population (using equations 1 and 2) in which the pavement roughness, vehicle type, vehicle load and speed play prominent roles. Information on the parameters needed for input into these equations should be collected from actual roads.

Table 1 Proposed percentiles of speed spectrum for calculation of dynamic tyre loads and pavement response for different road categories

Road category	Dynamic tyre load calculation	Pavement response calculation	
		Strain and deflection calculations	Stress calculations
A	95th	5th	95th
B	90th	10th	90th
C	80th	20th	80th
D	50th	50th	50th

The higher percentiles of the speed spectrum should be used to calculate the tyre loads and the pavement stress response parameters, while the lower percentiles of the speed spectrum should be used to calculate the pavement strain and deflection response parameters at speed. This is to ensure that conservative estimates of the specific response parameters are used. The recommended percentiles are shown in table 1 for each of the road categories.

To prevent the development of undue dynamic loads due to deterioration of pavement roughness through the life of the pavement, more frequent surface improvements with the aim of correcting the pavement roughness may be considered in the design. This will influence the rate at which pavement roughness deteriorates and the rate at which the dynamic tyre loads grow for the specific pavement.

Design traffic and pavement class

Design traffic and pavement class are concerned with the classification of pavements and traffic for structural design purposes. Determining the various equivalent loads for the pavement design is covered in this section.

The traffic classes used in TRH4 do not change with the new approach. However, the approximate vehicles per day per lane shown in the classification may increase due to higher tyre loads generated by moving dynamic loads. The tyre load population generated (equations 1 and 2) is used as the input to these calculations, causing the effect of the dynamic loads to be incorporated in the E80s calculated. The E80s are calculated using the standard load equivalence equation and an appropriate relative damage exponent. The E80 growth rates may be increased to cater for the deteriorating pavement roughness and concurrent increase in dynamic component of the tyre loads. The annual traffic loading may be calculated in a spreadsheet using different pavement roughness indicators for each analysis year. If major changes in any of the parameters used to calculate the tyre load spectrum are expected during the life of the road, a new tyre load spectrum should be calculated using the new input parameters.

Structural design and pavement type selection

Structural design and pavement type selection is concerned with pavement design methods, behaviour of different pavement types, factors influencing pavement layer selection, and the catalogue design method. The pavement design methods and behaviour of different pavement types are affected by the proposed method. The SAMDM is used with a linear elastic multi-layer analysis program and the selected percentile tyre load for the road category selected (table 1). Stresses, strains and deflections are calculated using this tyre load and the resulting response parameters are converted to a quasi-dynamic response parameter by application of the appropriate equations (Steyn 2001). Finally, the calculated response parameters are used with the SAMDM transfer functions to determine an estimated design life for the specific pavement. The expected pavement roughness deterioration should be evaluated again, as sharp deteriorations in the roughness may lead to accelerated pavement deterioration due to increases in dynamic load components.

Input data and assumptions for examples

Three examples of the process are shown for illustration. Certain assumptions are made as would normally be required. In the first example the standard current TRH4 based analysis is performed (static analysis – SA). In the second example the effect of tyre load population is incorporated into the design (TLP), while the third example also includes the effect of the moving tyre load incorporated into the example (Tyre load-speed effect – TLSE). The structural capacity of a specific pavement structure under the calculated traffic demands is calculated. No changes are made to the layer thicknesses to compensate for the different load spectra, and different bearing capacities are therefore calculated for the same pavement structure. These bearing capacities are all valid bearing capacities for the specific pavement structure, although they resemble different points on the distribution of possible bearing capacities for the pavement structure.

The pavement structure used for each of the examples consisted of a 40 mm asphalt surfacing, 150 mm crushed stone (G2) base and a 250 mm cemented natural gravel (C3) subbase. The selected layer is 500 mm natural gravel (G6). In TRH4 (1996) this pavement is classified as a category A road with a design traffic of 1,0–3,0 million E80s per lane (ES3).

The assumptions and input data indicated in table 2 were used in the analyses. The tyre load population calculated using equations 1 and 2 is shown in figure 2. A summary of the input data for the analyses is shown in table 2. Only those parameters that were different between the three analyses are shown. The percentile values used for the pavement response analyses are also indicated.

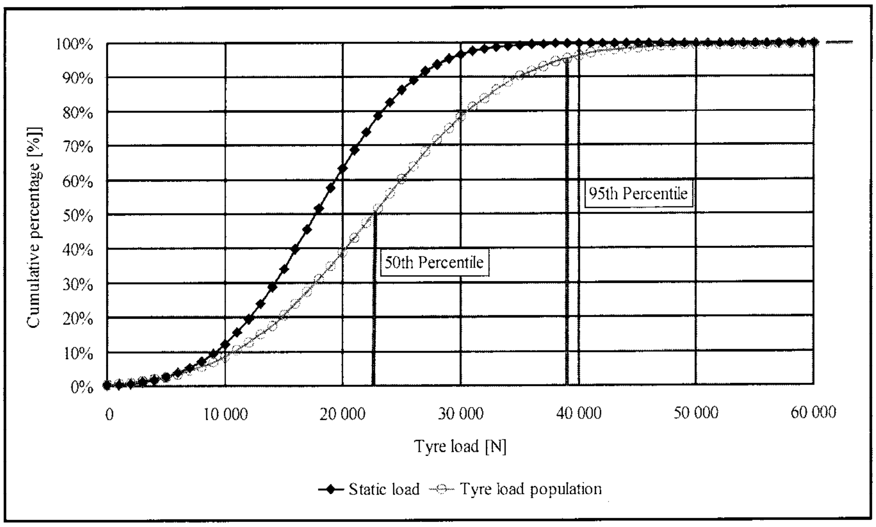

Figure 2 Tyre load populations developed using equations 1 and 2 and the input data in table 2

Analysis results

The results from the analyses are shown in table 3. All the analyses indicated the C3/EG4 layer as the most critical layer in the structure. The expected lives for the G2 base layer, G6 selected layer and in-situ subgrade were relatively high for all three analyses, indicating that these layers are well protected. The calculated life for the pavement structure as a whole was the longest under the static load (SL) and the shortest under the tyre load population (TLP) analysis. The asphalt surfacing showed a shorter expected life under the TLP analysis than under the tyre load-speed effect (TLSE) analysis, and the longest expected life under the SL analysis. The expected lives of the G2 base decreased under the TLP analysis but increased substantially under the TLSE analysis. The traffic demand under the SL analysis is for an ES10 traffic class, under the TLP analysis for an ES1 traffic class and under the TLSE analysis for an ES3 traffic class.

It appears as if the effect of the TLP analysis is to decrease the expected lives of all the layers, while the expected lives of the various layers increase again under the TLSE analysis. The decrease under the TLP population analysis is to be expected as the tyre load is increased. The increase in expected life under the TLSE analysis for the granular layers is mainly because of a reduction in the deviatoric stresses in the layer when the stresses are converted to equivalent dynamic stresses using the empirical equations. The increase in expected life for the asphalt layer under TLP analysis is due to a decrease in horizontal strain at the bottom of the asphalt layer when the static strain is converted to an equivalent dynamic strain. The increase in expected life for the C3/EG4 subbase layer between TLP and TLSE analyses is due to a decrease in the horizontal strains and a decrease in the deviatoric stress (during the granular phase) in this layer. A full discussion on the results is given in Steyn (2001).

When the complete pavement is evaluated it appears as if the use of TLSE conditions (both tyre load population and equivalent dynamic responses) decreased the expected pavement life by one traffic

Table 2 List of input data and assumptions that differed between the three analyses performed (full input data is provided in Steyn 2001)

TRH4 section	**Static load (SL) analysis**	**Tyre load population (TLP) analysis**	**Tyre load-speed effects (TLSE) analysis**
Section 1: Design philosophy and process	Not applicable	Functional service level dictates a maximum speed of 100 km/h and a typical pavement roughness level of 1,8 HRI. Life cycle strategy is selected to ensure a terminal pavement roughness of 2,7 HRI.	
Section 4: Design traffic	E80/heavy vehicle – 4,3[1] Static axle load distribution. Average Daily E80 (ADE) = 86,6[2]. E80 growth rate – 4 per cent. $AADE_{initial}$ – AADE – 86,6 Cumulative factor = 11 303.E80 total = $AADE_{initial}$ x 11303 = 978 839 E80s. Road category ES1	Average GVM = 137,2 kN (Rigid); 391,9 kN Articulated; 461,1 kN Interlink. Average number of tyres per vehicle = 6 (Rigid); 22 (Articulated); 26 (Interlink). 95th percentile of the speed population = 99 km/h. Average percentage load on vehicles = 75 per cent. Average tyre load for tyre load population = 22,9 kN (Rigid); 17,9 kN Articulated; 17,8 kN Interlink. Coefficient of Variation of tyre load population = 0,369% (Rigid); 0,531% Articulated; 0,571% Interlink Average Daily E80 (ADE) = 164,2 (new pavement); 170,9 (terminal condition)[3]. E80 growth rate = 4 per cent. $AADE_{initial}$ = AADE = 164,2 (new pavement); 170,9 (terminal condition). Cumulative factor = 11 303. E80 total = $AADE_{initial}$ x 11 303 = 1 855 952 E80s; 1 931 683 E80s (terminal condition). Road category ES3.	
Section 8: Pavement type selection and structural design			5th percentile speed = 23 km/h. 95th percentile speed = 99 km/h.

[1] The number of E80s per heavy vehicle for the static conditions are calculated based on the vehicle type distribution and the load levels per vehicle assumed for this example.

[2] The average daily E80 is calculated assuming 20 of the group of vehicles (1 rigid, 2 articulated and 1 interlink) to be using the road per day.

[3] The same group of vehicles as in footnote 2 have been used, but with the dynamic tyre load population for calculating these average daily E80 values.

Table 3 Expected lives calculated using static E80 tyre loads and static response analysis

Layer	Static load analysis (SL)		TYRE load population analysis (TLP)		Tyre load-speed effects analysis (TLSE)	
	Expected life [E80s]	Traffic class	Expected life [E80s]	Traffic class	Expected life [E80s]	Traffic class
Full life	7,99 million	ES10	0,56 million	ES1	2,41 million	ES3
Asphalt surfacing	3,21 million	ES10	0,27 million	ES0,3	9,71 million	ES10
Crushed stone base	> 100 million	ES100	6,38 million	ES10	> 100 million	ES100
Cemented gravel subbase	7,99 million	ES10	0,56 million	ES1	2,41 million	ES3
Natural gravel subbase	> 100 million	ES100	> 100 million	ES100	> 100 million	ES100
Subgrade	> 100 million	ES100	> 100 million	ES100	> 100 million	ES100

class (ES10 to ES3) for the conditions under which the analyses were performed. This equates to a decrease of 5,6 million E80s in structural capacity for the specific circumstances. It appears as if the effect of moving dynamic loads on the analysis of a pavement structure can be significant. Obviously, the combination of pave-ment layers and the specific tyre load popu-lation affect the magnitude and the results cannot be extrapolated to all conditions.

It is recommended that a sensitivity analysis be performed during the design process using different road classes and tyre load percentiles and pavement response percentiles. Such an analysis should indicate the range of expected pavement lives more accurately. In the example cited the 50th percentile tyre load is for instance only 22,5 kN. Using this tyre load and the 50th percentile pavement response option would lead to an increase in expected pavement life from a class ES3 to a class ES100. Sound engineering judgement of the specific problem is needed (as for all calculations of expected pavement life) before a final decision on the life of the pavement is made.

CONCLUSIONS

Tyre loads

- The average tyre load is a function of the Gross Vehicle Mass of the vehicle, while the Coefficient of Variation is a function of the vehicle speed and pavement roughness.
- Tyre load consists of a static and a dynamic load component of which the static load component is directly related to the GVM of the vehicles that use the pavement and the dynamic load component is directly related to the vehicle speed, vehicle type, GVM, load and pavement roughness.
- The tyre load distribution of a population of heavy vehicles can be described as normal.

Pavement response

- Load magnitude has a dominant effect on the calculated stresses while load speed has a dominant effect on the calculated strains and deflections in the pavement when using moving constant load analyses.
- The positions of maximum load application and maximum stress response at different depths in the pavement are not similar under a moving load. The distance between these positions increases with increased load application speeds. This distance lag is mainly due to the mass inertia properties of the pavement structure.

Guidelines

- The use of percentile values of the dynamic tyre load population rather than an equivalent static 80 kN axle load in pavement response analyses causes significantly different pavement responses.
- The use of moving dynamic loads and equivalent dynamic pavement response parameters in pavement analyses is possible.
- The use of a range of percentile values and input data values can assist in determining a population of pavement response values from which a better understanding of the vehicle–pavement interaction under all conditions can be extended.
- Moving constant and moving dynamic loads can affect pavement response analyses significantly and should be applied using good engineering judgement.

RECOMMENDATIONS

Several issues were identified as requiring further investigation. These issues are summarised here.

Vehicle-based assumptions

- The issue of changing vehicle characteristics with time and use that may influence the simulation of vehicle response to pavement roughness should be investigated to determine the extent of its influence on the calculated tyre load populations.
- The effect of incorporating a combination of both steel and air suspension when determining the tyre load populations should be addressed. The effect of using air suspension on the pavement is generally to lower the dynamic loads on the pavement and therefore a lower coefficient of variation can be assumed when incorporating air suspension.
- The effect of vehicles travelling around corners and on inclines should be addressed as this affects the contact stresses exerted on the surfacing layers of the pavement. Significant uphill and downhill travel will cause the speeds at which vehicles travel to fluctuate, affecting the tyre load population and elastic pavement deflection.

Pavement-based assumptions

- The use of the current SAMDM transfer functions for pavement structures in the analysis of moving tyre loads, leads to transfer functions based on static analysis of slow-moving tyre loads to be used to calculate the effect of moving dynamic loads on a pavement structure. Frequency-dependent materials such as asphalt are obviously influenced by this assumption more critically than non-frequency-dependent materials such as granular materials. Empirical equations may be developed to convert static response parameters to equivalent dynamic response parameters (Steyn 2001).
- The effects of risk and reliability in pavement design have not been addressed in this paper. It is important to have an understanding of the effect of reliability of input data on the output of the pavement design process.

References

DADS 1997. Dynamic analysis and design system. User's guide. Revision 8.5. Dynamic Interaction of Vehicle & Infrastructure Experiment: Asia-Pacific Concluding Conference, Melbourne, Australia: ARRB-TR.

Gilliomee, C L 1999. Simulation report: heavy vehicle load histories. Pretoria: Land Mobility Technologies (Pty) Ltd (Report nr R1/00015/1, Issue 1).

Jooste, F J 1999. Personal communication on axi-symmetric finite element programme based on Owen and Hinton (1980). Pretoria: CSIR. (Consultant.)

Steyn, W J vd M 2001. Considerations of vehicle-pavement interaction for pavement design. PhD thesis, University of Pretoria, Pretoria.

Steyn, W J vd M & Fisher, C 1997. Report of survey on truck suspension and tyre characteristics: Beitbridge – 2/3 December 1997. Pretoria: Division for Roads and Transport Technology, CSIR. (Technical Report TR-97/048.)

Steyn, W J vd M & Visser, A T 2000. Incorporating moving dynamic tyre loads in pavement design and analysis. *Proceedings,* South African Transport Convention, Pretoria.

Sweatman, P F 1983. A study of dynamic wheel forces in axle group suspensions of heavy vehicles. Victoria: Australian Road Research Board. (Special Report 27.)

Theyse, H L, De Beer, M & Rust, F C 1996. Overview of the South African Mechanistic Design Method. Pavement design, management and performance. Washington, DC: Transportation Research Board, National Research Council (Transportation Research Record; 1539), pp 6–17.

TRH4 1996. Structural design of flexible pavements for interurban and rural roads. Pretoria: Committee of Land Transport Officials (COLTO), Department of Transport. (Technical Recommendations for Highways; TRH4.)