# Proposed method to determine the horizontal shear between the interface of rib and block slab systems

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**M Gohnert**

***A method is proposed to determine the horizontal shear between the interface of rib and block slab systems. The horizontal shear is determined by calculating the change in the tension force across a segment due to flexural stresses. This approach differs from prevailing methods which base the horizontal shear as a function of the mid-span ultimate force. Equations are developed for both cracked and uncracked sections. Experiments were also performed to validate the proposed equations.***

MITCHELL GOHNERT graduated in 1985 with a BSc in civil engineering and in 1987 with an MEM (Master of Engineering Management) at Brigham Young University, Utah. He commenced employment with LTA Civil Designs and later joined Civemech, a subsidiary of Murray and Roberts. In 1990 he joined the Department of Civil Engineering at the University of the Witwatersrand. In 1995 he completed his PhD at the same university.

## INTRODUCTION

In SABS 0100 (South African Bureau of Standards 1992) a method is prescribed in section 6.4.4to determine the horizontal shear. This method is similar in many respects to BS 8110 (British Standards 1997) and the alternative method of ACI 318 (American Concrete Institute 1995).
As stated in 6.4.4.1:

> The interface of the precast and in-situ components occurs in the tension zone or in the compression zone affecting the horizontal shear force due to design ultimate loads so that this shear force is either: (a) where the interface is in the compression zone: the compression from that part of the compression zone above the interface, calculated from the ultimate bending moment; or (b) where the interface is in the tension zone: the total compression (or tension) calculated from the ultimate bending moment.

The code further states in section 6.4.4.2:

> The average horizontal design shear stress is calculated by dividing the design horizontal shear force by the area obtained by multiplying the contact width by the beam length between the point of maximum positive or negative design moment and the point of zero moment. The average horizontal design shear stress should then be distributed in proportion to the vertical design shear force diagram, to give the horizontal shear stress at any point along the length of the composite component. The maximum horizontal design shear stress $v_h$ should be less than the appropriate value in table 42.

The original specification on horizontal shear was changed in March of 1998 because a significant difference existed between the South African code and BS 8110. Prior to the change, the last sentence of clause 6.4.4.2 read:

> The average horizontal design stress $v_h$ should be less than the appropriate value in table 42.

The word 'average' was changed to 'maximum'.

In the past, many designers had assumed that the shear transferred is an average force over the length of the contact area; no attempt was made to distribute this force in accordance with the shape of the shear diagram. Since few publications gave guidance in this respect, design procedures followed composite concrete/steel design techniques (Wang & Salmon 1973; MacGregor 1988). The slip between composite concrete and steel is relatively large because of the difference in stiffness of each material. This causes a redistribution of force along the length of the contact surface and therefore an average stress can be assumed. This same rationale was also applied to a composite member where both materials are concrete. However, as of late, many codes have changed. In ACI 318-95, a commentary note was included stating that in composite construction, where both materials are concrete, the slip along the interface is small and therefore the redistribution of stress is limited and confined to a small region. If this is correct, an average stress cannot be assumed (due to a lack of redistribution) and the design stress should be taken as the maximum stress and proportional to the shape of the shear diagram..

As a result of the change, the South African code is now compatible with many other international codes. However, the change had serious ramifications on the design of composite members. For example, in a simply supported beam subjected to a uniformly distributed load, the maximum shear stress at the support is double the average stress. The horizontal shear is therefore, potentially, the 'bottle neck' of the design since the design must now account for a much higher shear stress.

The rib and block floor system is widely used in South Africa and it is estimated that at least half of the floor slabs are constructed with this system. Residential structures and office blocks comprise the bulk of the market. In general, this floor system has worked well with only a few recorded failures in horizontal shear. This type of failure seems to be rare and for those few cases that are publicised, usually the cause of failure is poor construction practice or the use of inferior building materials.

As a result of the change to the code, the economy of the rib and block slab system has been in question. This has spurred research in this area since prior designs appear to be adequate with regards to horizontal shear. Both the assessment of the horizontal shear and the capacity of the interface are subjects to be questioned. This paper, however, only considers the assessment of the horizontal shear.

The equations derived here are applicable to a simply supported beam under the influence of a uniformly distributed load, bent in flexure. In addition, the precast member and the contact interface are assumed to be located in the tension zone. This is generally the case with rib and block floor slabs and therefore directly applicable to this type of system. Since the concrete may or may not be cracked at the location of critical shear, equations are developed for both cases.

# HORIZONTAL SHEAR AND THE SABS 0100 METHOD

A bending moment will cause a stress distribution as depicted in figure 1(a). If we cut along the line b–c of the segment, the horizontal shear force along that plane is the difference between the resultant forces (see figure 1b).

Summing forces in figure 1b

$$\Sigma F = F - (F + dF) - V_h = 0 \qquad (1)$$

or,

$$V_h = F - (F + dF) \qquad (2)$$

where $V_h$ is the horizontal shear force and F is the resultant force of the stress distribution

The length of the differential segment in figure 1 is $dx$. If we consider a much larger segment equal to half the span length (ie, from the support to mid-span – see figure 2) our definition of the horizontal shear is simplified. In a simply supported beam, the bending stress at the support is zero and therefore the resultant force is zero. Summing forces in figure 2,

$$\Sigma F = V_h = F \qquad (3)$$

The shear force is thus equal to the resultant force at mid-span.

The average shear stress is then determined by dividing the shear force by the area of the contact surface.

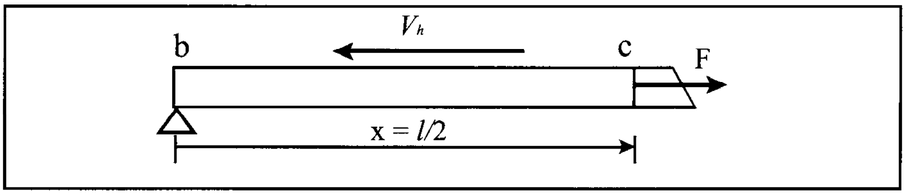


***Figure 2 Horizontal shear force and resultant force in a beam segment of length x***

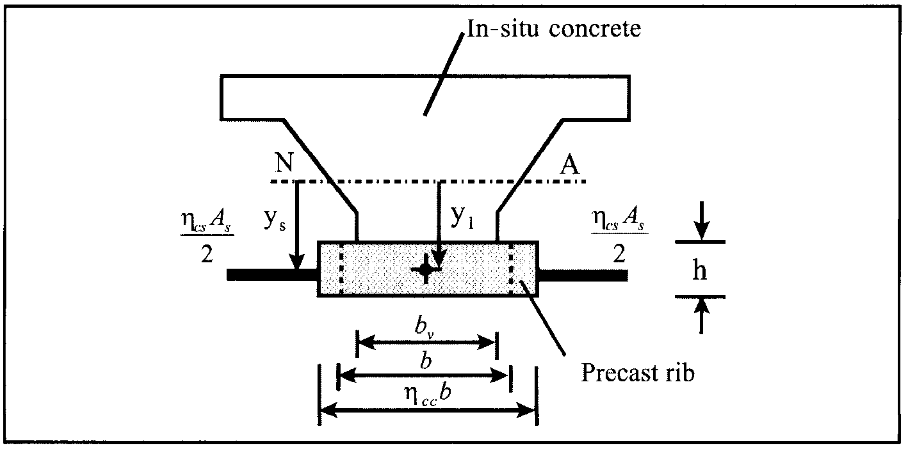


***Figure 3 Transformed section of a precast rib***

$$v_h = \frac{2V_h}{b_v l} \qquad (4)$$

Where $l$ is the span length and $b_v$ is the width of the contact surface.

Distributing this stress according to the shape of the vertical shear diagram, the horizontal shear stress can be determined at any point along the length of the span.

$$v_h = \frac{-8V_h}{b_v l^2} x + \frac{4V_h}{b_v l} \qquad (5)$$

In a simply supported beam, the maximum shear stress will occur at the support. It is therefore inevitable that the design shear stress will be calculated at x = 0.

$$v_h = \frac{4V_h}{b_v l} \qquad (6)$$

Traditionally, the horizontal shear ($V_h$) is determined as the mid-span force calculated from the ultimate bending moment. Usually this mid-span force is the ultimate strength of the reinforcing or prestressing steel. This is clearly a conservative approach. The shear force cannot exceed this value since the member, at ultimate load, is subjected to impending flexural failure. The prevailing method of design is therefore based on the flexural capacity of the member. The method is simple and safe, but oddly not a function of the load, span length or boundary condition all of which influence the magnitude of the horizontal shear.

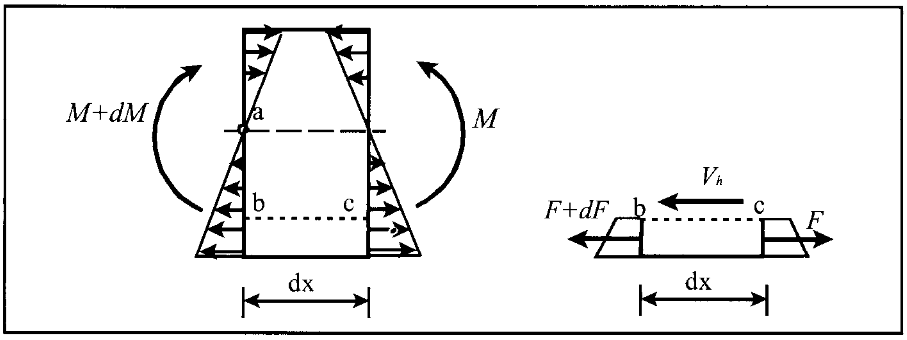


***Figure 1(a) Bending stress distribution in a segment; (b) Resultant forces and the horizontal shear force along the cut surface b–c***

# PROPOSED METHOD TO DETERMINE THE HORIZONTAL SHEAR

## Horizontal shear stress of an uncracked section

In the composite member, three material properties will be present the in-situ concrete, the precast concrete and the reinforcing steel. An analysis would require that at least two materials are transformed into the third. To do so, two modular ratios are defined:

$$\eta_{cc} = \frac{E_{rib}}{E_{in\text{-}situ}} \qquad (7)$$

$$\eta_{cs} = \frac{E_{steel}}{E_{in\text{-}situ}} \qquad (8)$$

The in-situ concrete in a rib and block system is usually irregular in shape. For this reason, it is easier to transform the precast rib and steel into the same material as the in-situ concrete. The transformed section is illustrated in figure 3. The gross moment of inertia ($I_g$) is based on this transformed section.

Since the section is assumed to be uncracked, the stress in the precast member is composed of two parts the stress in the concrete and the stress in the steel.

$$f_c = \frac{\eta_{cc} M y_1}{I_g} \qquad (9)$$

$$f_s = \frac{\eta_{cs} M y_s}{I_g} \qquad (10)$$

The term $f_c$ is the average stress in the precast member due to flexure, $f_s$ is the stress in the steel due to flexure, $y_1$ is the distance from the neutral axis to the centre of the precast member and $y_s$ is the distance from the neutral axis to the steel centroid.

The resultant force of each material is simply the stress multiplied by the cross-sectional area of each material. The total resultant force is the sum of the two.

$$F = \frac{\eta_{cc} M y_1 bh}{I_g} + \frac{\eta_{cs} M y_s A_s}{I_g} \quad (11)$$

The length of the segment extends from the end of the beam (where the flexural stress is zero) to some point x along the span. By making this assumption, equation 3 applies the resultant force at the support is zero and therefore the horizontal shear is equal to the resultant force calculated at some point x along the span.

$$V_h = F = \frac{\eta_{cc} M y_1 bh}{I_g} + \frac{\eta_{cs} M y_s A_s}{I_g} \quad (12)$$

The average shear stress over a distance x is equal to the shear force divided by the contact interface.

$$v_h = \frac{V_h}{b_v x} = \frac{M}{I_g b_v x}(\eta_{cc} y_1 bh + \eta_{cs} y_s A_s) \quad (13)$$

The bending moment, for a simply supported beam subjected to a uniformly distributed load, is given in equation 14.

$$M = \frac{wlx}{2}\left(1 - \frac{x}{l}\right) \quad (14)$$

This equation is substituted into equation 13:

$$v_h = \frac{wl(1 - \frac{x}{l})}{2 I_g b_v}\left[\eta_{cc} y_1 bh + \eta_{cs} y_s A_s\right] \quad (15)$$

where x is valid within the limits of $0 \leq x \leq l/2$

Equation 15 solves for the average horizontal stress over a length $x$. However, at $x = 0$, the stress is no longer the average, but the maximum horizontal shear stress at the support.

$$v_h = \frac{wl}{2 I_g b_v}[\eta_{cc} y_1 bh + \eta_{cs} y_s A_s] \quad (16)$$

Upon close inspection of equation 16, the term $[\eta_{cc} y_1 bh + \eta_{cs} y_s A_s]$ is the first moment of area which can be defined by the symbol. Furthermore, the symbol $wl/2$ is equal to the vertical shear force of a simply supported beam subjected to uniformly distributed load. Substituting these symbols into equation 16,

$$v_h = \frac{VA\bar{y}}{I b_v} \quad (17)$$

Equation 17 is the original form of the horizontal shear equation used in CP 110 (British Standards 1972). Although different symbols are used, the equations are essentially the same. The only difference is the shear force ($V$) – CP 110 is based on working loads and equation 16 is based on ultimate loads.

Most international codes are based on ultimate limit state theory. Equations were consequently developed to characterise the ultimate state of stress under factored loads. Despite the fact that an ultimate condition is assumed, we still utilise an elastic distribution of factored moments and shears. This apparent inconsistency is only tolerated because it is simple and a practical way of determining a state of stress. The same is true of horizontal shear equations. Although an elastic equation is not an exact representation, they are the simplest and most practical way of estimating the horizontal shear (Loov 1994).

## Horizontal shear stress of a cracked section

The derivation for the cracked case is similar to the uncracked case. The first step is to solve for the transformed cracked section (see figure 4). The cracked moment of inertia is based on this section.

For the cracked case, the tension stress is assumed to be resisted solely by the reinforcement or pretensioned steel. The stress in the steel is determined by equation 18.

$$f_s = \frac{\eta_{cs} M y_s}{I_{cr}} \quad (18)$$

The resultant force is equal to the stress multiplied by the area of steel.

$$F = \frac{\eta_{cs} M y_s A_s}{I_{cr}} \quad (19)$$

Similar to the uncracked derivation, the length of the beam segment extends from the support to some point x along the span. The resultant force at the support is equal to zero. Therefore, equation 3 applies and the shear force is equal to equation 19.

$$V_h = F = \frac{\eta_{cs} M y_s A_s}{I_{cr}} \quad (20)$$

The average shear stress, over a distance $x$, is solved by dividing the shear force by the area of the contact interface.

$$V_h = \frac{\eta_{cs} M y_s A_s}{I_{cr} b_v x} \quad (21)$$

The moment equation (equation 14) is then substituted into the above equation.

$$v_h = \frac{\eta_{cs} wl(1 - \frac{x}{l}) y_s A_s}{2 I_{cr} b_v} \quad (22)$$

Solving for the stress at $x = 0$, the stress is no longer the average, but the maximum horizontal shear stress at the support.

$$v_h = \frac{\eta_{cs} wl y_s A_s}{2 I_{cr} b_v} \quad (23)$$

Similar to equation 16, the term is the first moment of area and is the vertical shear. This equation can also be reduced to the general form of equation 17.

## EXPERIMENTS

A total of six composite beams were tested in flexure to determine validity of the above equations. The basic dimension and load arrangement of each of the test beams are given in figure 5.

A uniformly distributed load is approximated by four point loads as illustrated. The contact width was reduced in total by 50 mm by wedge-shaped formers placed on either side of the precast member. This reduction in the contact width resembles the effect of the blocks that are placed on either side of the precast rib.

The precast member was lightly brushed on the contact surface and no shear links were provided. The average

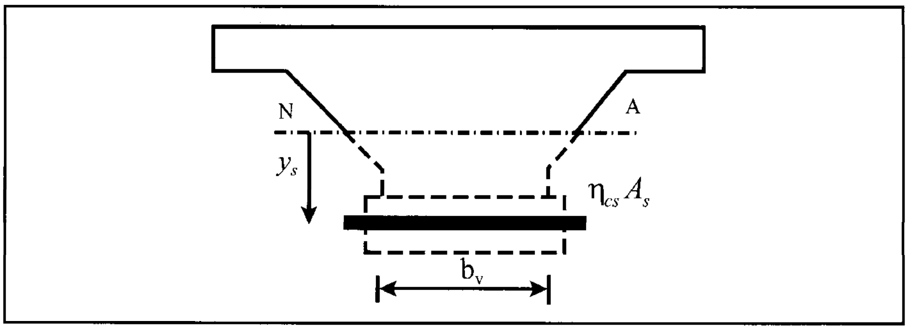


*Figure 4 Transformed cracked section*

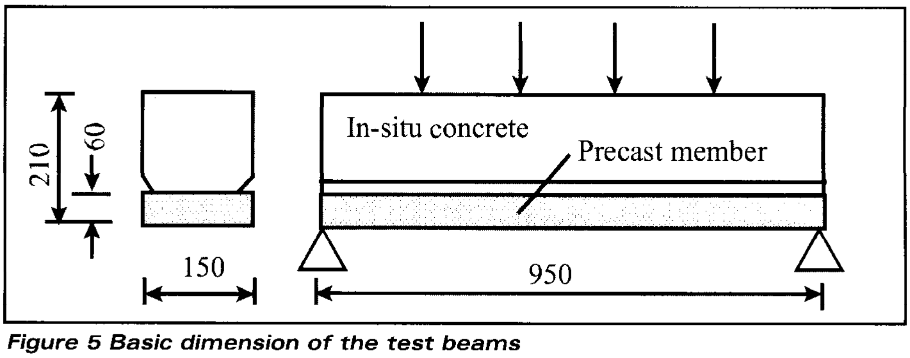


*Figure 5 Basic dimension of the test beams*

*Table 1: Prestress and concrete information of the six test beams and failure loads.*

| Beam number | Concrete strength of precast members (MPa) | In-situ concrete strength (MPa) | Number and height of strands above the soffit of the precast member | Strand diameter (mm) | Yield strength of strand (MPa) | Failure load (kN) |
|---|---|---|---|---|---|---|
| A1 | 42,7 | 31,0 | 3 @ 37 mm<br>2 @ 20 mm<br>2 @ 10 mm | 4 | 1700 | 110 |
| A2 | 42,7 | 31,0 | 3 @ 37 mm<br>2 @ 20 mm<br>2 @ 10 mm | 4 | 1700 | 94 |
| A3 | 42,7 | 16,6 | 1 @ 35 mm<br>2 @ 15 mm | 4 | 1700 | 60 |
| A4 | 42,7 | 16,6 | 4 @ 40 mm<br>4 @ 15 mm | 4 | 1700 | 45 |
| A5 | 42,7 | 20,8 | 1 @ 35 mm<br>2 @ 15 mm | 4 | 1700 | 75 |
| A6 | 42,7 | 20,8 | 1 @ 35 mm<br>2 @ 15 mm | 4 | 1700 | 94 |

height of roughness was measured as 0,94 mm (an average of 15 readings).

The precast members are prestressed with a varying number of 4 mm diameter strands. The prestressing information and concrete strengths are given in table 1.

In prior tests (not recorded here) the mode of failure was vertical shear in the in-situ concrete. The shear crack followed a typical shear pattern – shear cracks extending from the support at an incline from 30 to 40 degrees. In all of the preliminary tests, vertical shear failure precipitated the horizontal shear failure. To isolate the horizontal shear, shear links were placed in the in-situ concrete – 4 mm diameter high tensile steel spaced at 100 mm. The mode of failure then switched to a horizontal shear failure as illustrated in figure 6 and the failure loads are given in table 1. This failure was dramatic and seemingly instantaneous. The failure line extends from the support, along the contact interface and then vertically between the shear reinforcement. Furthermore, the failure line along the contact interface did not remain localised or limited in extent (as suggested by ACI 318), but ranged from 50 to 332 mm in length.

Twelve additional tests were also done to determine the horisontal shear strength along the contact interface. The 'push-off' method (Lam *et al* 1998; Choi *et al* 1999) was used to determine this value. The test rig is illustrated in figure 7. The rib is braced and a ramping load is applied to the in-situ concrete until failure occurs.

The dimensions of these test beams are the same as illustrated in figure 5, but the length of the beams was reduced to 750 mm. A set of six beams were tested at 22,8 MPa and the other six at 31,4 MPa (in-situ concrete strength) to project the horizontal shear strength at different in-situ concrete strengths. The results of these tests are given in table 2.

From table 2, a linear equation was fitted to the data to determine the horizontal shear strength ($v_{nh}$) at different in-situ concrete strengths:

$$v_{nh} = 0{,}025 f_{cu} + 0{,}377 \qquad (24)$$

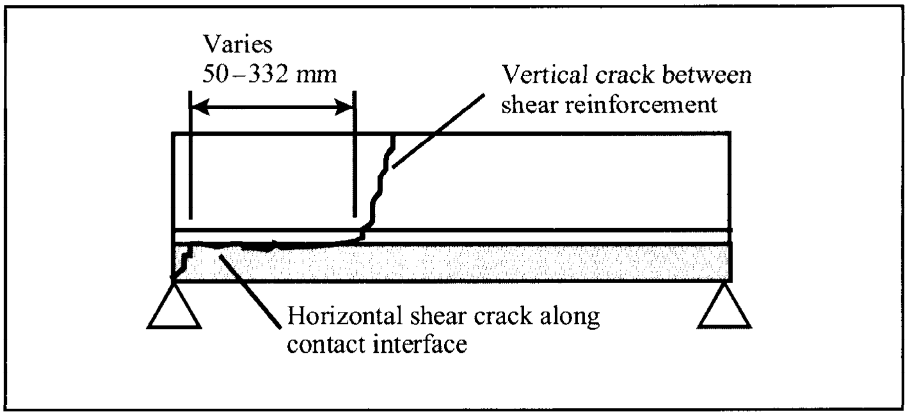


*Figure 6 Typical shear pattern in test beams*

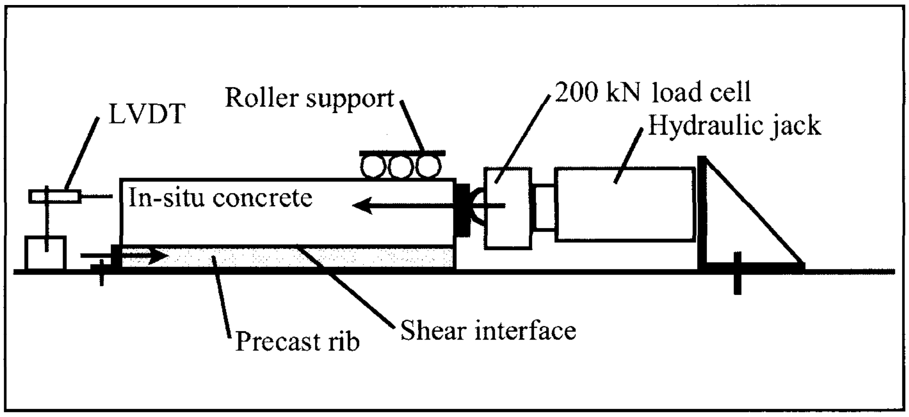


*Figure 7 Schematic of the test rig used in the 'push off' test*

*Table 2 Horizontal shear capacity of test beams*

| Beam number | Horizontal shear strength (MPa) | Concrete strength |
|---|---|---|
| B1<br>B2<br>B3<br>B4<br>B5<br>B6 | 1,05<br>0,69<br>1,03<br>1,25<br>0,79<br>0,87 | In-situ concrete strength: 22,8 MPa<br><br>Rib concrete strength: 41,7 MPa |
| B7<br>B8<br>B9<br>B10<br>B11<br>B12 | 1,24<br>1,27<br>0,94<br>1,15<br>1,11<br>1,26 | In-situ concrete strength: 31,4 MPa<br><br>Rib concrete strength: 41,7 MPa |

*Table 3 Comparison of experimental data, proposed equations and SABS 0100*

| Beam number | $V_h$ Equation 16 (MPa) | $V_h$ Equation 23 (MPa) | SABS 0100 (MPa) | Experimental equation 25 (MPa) |
|---|---|---|---|---|
| A1<br>A2 | 3,33<br>2,84 | 3,17<br>2,71 | 5,48<br>5,48 | 0,98 – 1,32<br>0,98 – 1,32 |
| A3<br>A4 | 1,84<br>1,40 | 1,67<br>1,33 | 2,35<br>6,26 | 0,62 – 0,96<br>0,62 – 0,96 |
| A5<br>A6 | 2,29<br>2,87 | 2,09<br>2,62 | 2,35<br>2,35 | 0,73 – 1,07<br>0,73 – 1,07 |

It should be noted that equation 24 has not been reduced to an appropriate confidence interval nor have any safety or material factors been applied. The objective is not to formulate a design curve, but to estimate the shear strength for a given in-situ concrete strength.

An inspection of the test data indicates a rather large scatter in the results. This is typical for these types of tests despite the care in constructing and testing the specimens. The standard deviation of table 2 is 0,169. It is, perhaps, more appropriate to express an estimate of the shear strength over a range.

$$v_h = 0{,}025 f_{cu} + 0{,}377 \pm 0{,}169 \qquad (25)$$

The proposed equations (equations 16 & 23) were then compared to the experimental data (equation 25) and the equations endorsed by SABS 0100. This comparison is given in table 3.

A sample calculation of the first row of table 3 is given in Appendix 1.

# CONCLUSIONS

The SABS 0100 method is conservative and safe, but at the expense of economy. Rib and block slab systems are not known to be problematic in horizontal shear despite the minimal contact area of the interface, yet the code has doubled the shear that one must account for. Theoretically, the code method is sound, but a refinement is possible. Table 3 implies that the proposed equations predict the horizontal shear better than current methods.

The majority of code equations are based on the ultimate strength of the prestressing strands or reinforcement. The horizontal shear will not exceed this value, since the member will fail in flexure at this stage. Oddly, the SABS 0100 method is not a function of the load, boundary conditions or span length. The proposed equations differ in a sense that the calculation is not based on the flexural capacity, but the horizontal shear is determined by calculating the change in the tension force across a segment and at the point of maximum shear. This gives a more accurate representation of the actual horizontal shear stress.

The experiments suggest that the horizontal shear stresses are not confined to a small region, but a failure mechanism is formed; and, in several cases, the failure line nearly reached mid-span.

This is in opposition to the commentary note of ACI 318-95, which states that the shear stresses are limited and confined to a small region. It appears that the ACI specification – and many others – is based on experiments which contain at least minimal steel across the contact interface.

The proposed method is similar to the old CP 110 method, but differs in a sense that the loads are calculated at the ultimate limit state and the cross-section is calculated as either cracked or uncracked. As illustrated by the above derivation, the method listed in SABS 0100 is a rudiment form of a more precise method given by equations 16 and 23.

The methods listed in SABS 0100 and BS 8110 are intended to give results that are similar to the old CP 110 method (Rowe *et al* 1985). However, as shown by Table 3, this is not the case. If the amount of reinforcing or prestressing steel is not proportionate to the applied loads, a disparity will exist.


### Acknowledgements

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# APPENDIX 1

The following is an example of the calculations for beam number 1 of table 3:

*Calculation parameters*

$f_{cu\,(rib)} = 42{,}7$ MPa
$f_{cu\,(in\text{-}situ)} = 31{,}0$ MPa
Failure load = 110 kN
$f_y = 1700$ MPa
Prestressing wires (4 mm diameter) - 3@ 37 mm, 2@ 20 mm and 2@ 10 mm
$A_s = 12{,}57$ mm$^2$
$A_s$(total) = 7(12,57) = 87,99 mm$^2$
$E_{rib} = 20 + 0{,}2(42{,}7) = 28{,}5$ MPa
$E_{in\text{-}situ} = 20 + 0{,}2(31{,}0) = 26{,}2$ MPa

$\eta_{cc} = 28{,}5/26{,}2 = 1{,}09$
$\eta_{cs} = 205/26{,}2 = 7{,}82$

*Equation 16 – uncracked section*

Centroid of the steel =

$$\frac{3(12{,}57)37 + 2(12{,}57)20 + 2(12{,}57)10}{87{,}99} = 24{,}43 \text{ mm}$$

Neutral axis =

$$\frac{150^2(135) + 1{,}09(150)60(30) + 7{,}82(87{,}99)24{,}43}{150^2 + 1{,}09(150)60 + 7{,}82(87{,}99)} = 101{,}48 \text{ mm}$$

$$I_g = \frac{150^4}{12} + 150^2(101{,}48 - 135)^2 + \frac{1{,}09(150)60^3}{12}$$
$$1{,}09(150)60(101{,}48 - 30)^2 + 7{,}82(87{,}99)(101{,}48 - 24{,}43)^2$$
$$= 124{,}6(10^6)\text{ mm}^4$$

$$v_h = \frac{110(10^3)}{2(124{,}6)(10^6)100}[1{,}09(71{,}48)150(60) + 7{,}82(77{,}05)87.99] = 3{,}33 \text{ MPa}$$

*Equation 23 – cracked section*

X = 36,97 mm
Neutral axis = 210 – 36,97 = 173,03 mm

$$I_{cr} = \frac{150(36{,}97)^3}{12} + 150(36{,}97)18{,}49^2 + 7{,}82(87{,}99)148{,}6^2 = 17{,}7(10^6)\text{ mm}^4$$

$$v_h = \frac{110(10^3)}{2(17{,}7)(10^6)100}[7{,}82(148{,}6)87.99] = 3{,}18 \text{ MPa}$$

*Equation 6 – SABS 0100 method*

$$v_h = \frac{4(1700)87{,}99}{1{,}15(100)950} = 5{,}48 \text{ MPa}$$