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Modelling soil stratification under triaxial loading using critical state soil mechanics

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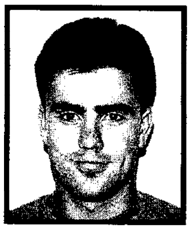

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This paper compares the performance of critical state constitutive modelling with laboratory triaxial test data from a classic series of tests on reconstituted Weald clay. Critical state models used for predicting the test results include the original Cam-Clay model, Cam-Clay incorporating the planer Hvorslev surface on the dry side of critical and the Modified Cam-Clay model. The purpose of the comparison is to illustrate the dangers of using constitutive relations and models in numerical analyses without a proper understanding of their fundamental assumptions and inherent limitations. It will be shown that Modified Cam-Clay performs satisfactorily under the specific conditions of standard triaxial compression testing and that it fits experimental stress paths closer than the original Cam-Clay model. However, it is of utmost importance to include the Hvorslev state boundary surface on the dry side of critical for both these models if realistic predictions are to be made for heavily overconsolidated soils.

INTRODUCTION

Throughout the past half century, a great deal of research and development has taken place on the behaviour of soils and on constitutive modelling of soil beha-viour. It can be stated that Critical State Soil Mechanics (CSSM) was born at the University of Cambridge in the 1950s with K H Roscoe, A N Schofield and C P Wroth. These men developed the first constitutive soil models based on critical state soil mechanics; they were the Granta Gravel and the original Cam-Clay models for idealised sand and clay behaviour. Since then a number of modifications and new models have seen the light based on observed material behaviour. These are used with great success in the analysis of geotechnical problems. Critical state models draw together concepts of compression, shear, yield and failure into a single unified frame-work. This unification of behavioural concepts provides the foundation for a proper understanding of soil behaviour and the tools required for numerical analysis of geotechnical problems.

For the purposes of this paper the following basic underlying assumptions apply:

- Only saturated clays are considered, a two-phased medium consisting of water and soil grains.
- Material strength is assumed to be purely frictional, ie no cementation or apparent cohesion, except as a result of pore water suction.
- The material is considered to behave like an ideal elasto-plastic continuum with states of stress and strain assumed to be uniform throughout the material except in the case of discontinuous slippage occurring under conditions of work softening and strain localisation.

The development of the critical state theories have progressed to include the effects of partial saturation, cementation, etc, but these are conceptually and mathematically much more complex and beyond the scope of this discussion.

STRESS AND STRAIN REPRESENTATION

States of stress and strain

A set of six independent effective stresses $(\sigma'_x, \sigma'_y, \sigma'_z, \tau_{xy}, \tau_{yz}, \tau_{zx})$ acting on an element of dimensions *(dx, dy, dz)* describes completely the current state of stress at a singular point in a soil mass or sample. These stress variables, however, are sensitive to the choice of reference axes (x:y:z) and their values will be affected by changing the orientation of the reference axes, ie the viewpoint, which makes them less than ideal for the purposes of studying the behaviour of soils.

By constructing Mohr's circles to represent all possible combinations of normal and shear stresses acting in any direction at the point under consideration, three independent principal effective stresses σ'_1, σ'_2, σ'_3 are found, which together with the orientations of these planes (principal planes of stress) describe the state of stress at the specified point. To this extent the current state of stress and the stress path (or map of the history of stress changes) can be represented as a function of the principal effective stresses or $M' = f(\sigma'_1, \sigma'_2, \sigma'_3)$. The value of M' in terms of σ'_1, σ'_2 and σ'_3 is now independent of the choice of reference axes and together with the orientations of the principal planes describe the 3D stress state completely. However, the orientations of the principal planes, and hence the directions of σ'_1, σ'_2 and σ'_3, may change as a result of loading, for example during one-dimensional unloading.

A more elegant way of describing M′ is by means of invariants of stress which are independent of both the choice of reference axes and of rotation of the principal planes of stress. Another advantage of using invariants is the ease of normalisation of stress paths onto reference planes to facilitate interpretation. Without going into the detailed mathematics of invariants these will simply be stated in a form appropriate to the study of soil mechanics. For a general 3D stress state there are three independent stress invariants; in terms of σ'_1, σ'_2 and σ'_3 they are

- Mean normal effective stress, a hydrostatic (equal all round) stress

$$p' = \tfrac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3) \quad (1)$$

- Deviator stress, a shear stress

$$q' = \frac{1}{\sqrt{2}}\left[(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2\right]^{1/2} \quad (2)$$

- An orientation angle α

The physical interpretation of these parameters is illustrated by figure 1

where $O'N' = \sqrt{3}p'$

$N'M' = \frac{\sqrt{6}}{3}q$

Note M = State of total stress corresponding to M'

$MM' = \sqrt{3}u$ separates states of total and effective stress

Similar arguments hold for describing the state of 3D strain and strain paths in terms of the strain invariants. Using the principal strains, ε_1, ε_2 and ε_3, these are

- Volumetric strain – a change in volume (compression/dilation)

$$\delta\varepsilon_p = \delta\varepsilon_v = (\delta\varepsilon_1 + \delta\varepsilon_2 + \delta\varepsilon_3) \quad (3)$$

- Deviatoric shear strain – a change in shape (distortion)

$$\delta\varepsilon_q = \frac{\sqrt{2}}{3}\left[(\delta\varepsilon_1 - \delta\varepsilon_2)^2 + (\delta\varepsilon_2 - \delta\varepsilon_3)^2 + (\delta\varepsilon_3 - \delta\varepsilon_1)^2\right]^{1/2} \quad (4)$$

- An orientation angle α

For the special case of axial symmetric loading, which applies during a standard triaxial test with σ'_2, σ'_3 and $\varepsilon_2 = \varepsilon_3$, the stress and strain invariants simplify to

- Mean normal effective stress

$$p' = \tfrac{1}{3}(\sigma'_1 + 2\sigma'_3) = \tfrac{1}{3}(\sigma'_a + 2\sigma'_r) \quad (5)$$

- Volumetric strain

$$\delta\varepsilon_p = (\delta\varepsilon_1 + 2\delta\varepsilon_3) = (\delta\varepsilon_a + 2\delta\varepsilon_r) \quad (6)$$

- Deviatoric shear stress

$$q' = (\sigma'_1 - \sigma'_3) = (\sigma'_a - \sigma'_r) \quad (7)$$

- Deviatoric shear strain

$$\delta\varepsilon_q = \tfrac{2}{3}(\delta\varepsilon_1 - \delta\varepsilon_3) = \tfrac{2}{3}(\delta\varepsilon_a - \delta\varepsilon_r) \quad (8)$$

- $\alpha = 0$, for both the stress and strain invariants

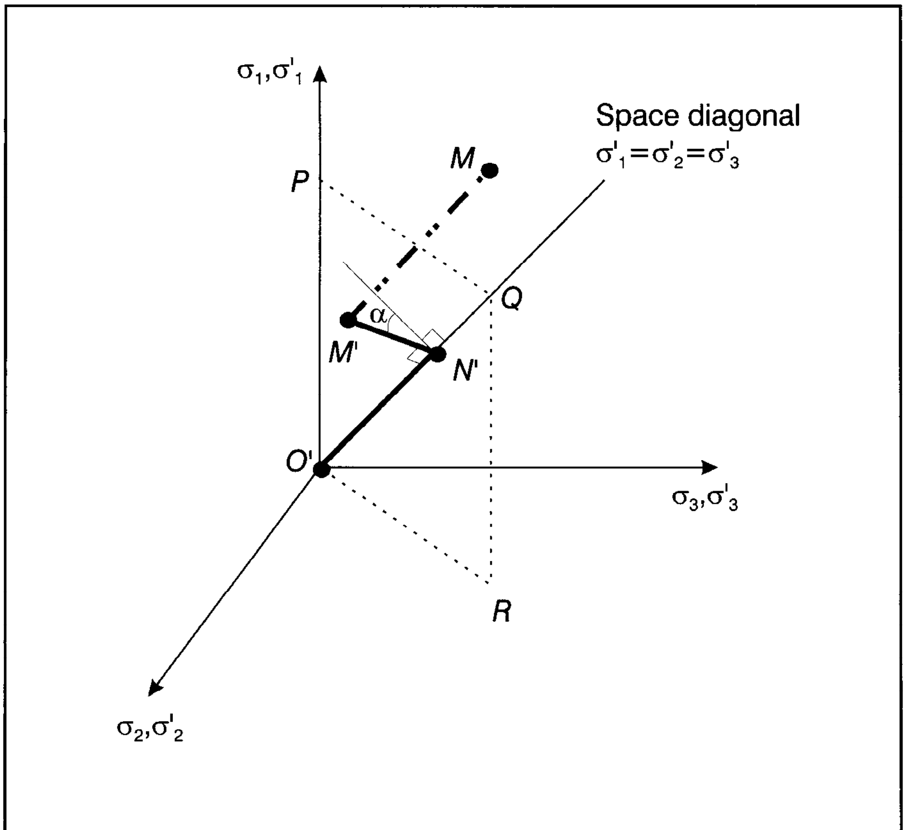


Figure 1** **Representation of the physical interpretation of the stress invariants p′, q′ and α

- The stress ratio $\eta = \frac{q'}{p'}$ becomes

$$\eta = \frac{3(\sigma'_1 - \sigma'_3)}{\sigma'_1 + 2\sigma'_3} = \frac{3(\sigma'_a - \sigma'_r)}{\sigma'_a + 2\sigma'_r} \quad (9)$$

where σ'_a = Axial effective stress acting on a triaxial specimen
σ'_r = Radial effective stress acting on a triaxial specimen
$\delta\varepsilon_a$ = Axial strain of a triaxial specimen
$\delta\varepsilon_r$ = Radial strain of a triaxial specimen

The use of triaxial stresses and strains, instead of principal stresses and strains, allows for a differentiation between states of compression (+q') and extension (–q'), often required during stress path testing. Incidentally, the deviator stress, q', is numerically equal to the applied ram stress.

Plotting load paths for triaxial compression and shear

It is common practice in critical state modelling to follow the progression of the change of stress and strain by plotting the stress invariants p′ and q′ together with the strain invariant, υ, the specific volume. Specific volume is numerically equal to 1 + e (void ratio) and relates to volumetric strain in the following manner:

$$\upsilon = 1 + e \quad (10)$$

$$\delta\varepsilon_p = \frac{-\delta\upsilon}{\upsilon} = \frac{-\delta e}{1+e} = \frac{-\delta V}{V} = \frac{\delta V_w}{V} \quad (11)$$

where V is the total volume of the specimen

δV_w is the volume of water expelled (+) in an increment of volumetric strain.

Conventionally the results of drained and undrained triaxial shear tests are represented using the deviator stress vs axial strain and either the volumetric strain (drained) or pore pressure (undrained) vs axial strain plots. Both the critical state and conventional representations will be given in this paper.

CRITICAL STATE SOIL MECHANICS

The constitutive modeller is faced with the basic problem of complexity vs practicality with regard to the following:

- *Complexity:* A model should be flexible enough to account for all or most of the following: saturated, partially saturated and dry conditions; isotropic and anisotropic soil properties and behaviour; and recoverable as well as permanent deformations (elastic and plastic responses).
- *Practicality:* In order to keep a model

simple and practical only the physical factors relevant to the particular problem at hand should be incorporated. Such a set of constraints, appropriate for the study of triaxial test data, may include saturated clay behaviour under normally consolidated or lightly overconsolidated states subject to axial symmetric loading.

The development of a constitutive model for soils should furthermore be based on physical observations of actual material behaviour under both laboratory and field loading conditions. The benefits of such a constitutive relationship that satisfies these requirements are as follows:

- It provides a basis for the interpretation and organisation of drained and undrained laboratory test data for various states of stress and deformation.
- It provides the means (in conjunction with appropriate computer codes) to perform effective and/or total stress analysis for a wide variety of earth structure problems for either transient- or static-type loading conditions.

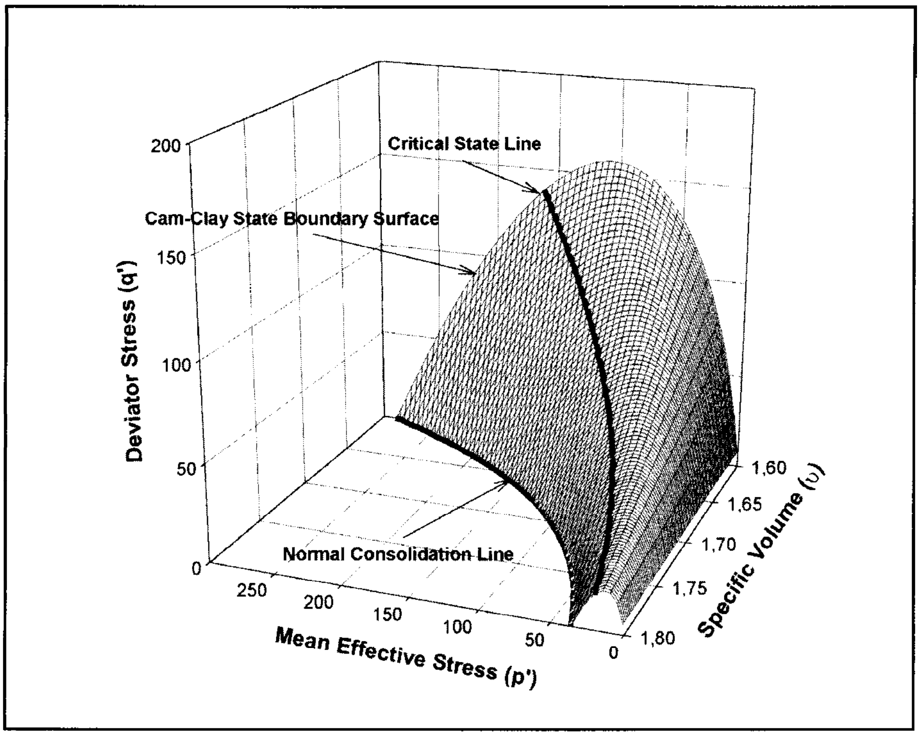
Basic assumptions relevant to constitutive modelling

In general assumptions are needed on five aspects of the elasto-plastic response of a continuum. For soils these are:

- *Elastic properties:* A description of the way in which elastic or recoverable deformations are to be described, for example linear isotropic, non-linear isotropic and non-linear anisotropic. Elastic deformations result from purely elastic behaviour prior to yield and from elastic strains, which occur simultaneously with plastic strains, during yield.
- *Yield surface:* The yield surface, also known as the state boundary surface (SBS), constitutes a boundary in general stress space between purely elastic and elasto-plastic behaviour. Intersections of elastic stress paths with the yield surface are known as yield curves and constitute purely elastic behaviour on the SBS. Plastic behaviour goes hand in hand with a change in the size of the yield curve.
- *Flow rule:* The flow rule describes the mode of plastic deformation that occurs when a soil is yielding, ie the relative magnitudes of the volumetric and shear components of plastic deformation. The flow rule is usually described by the plastic potential, which constitutes a surface normal to the plastic strain increment vector. Some common flow rules include:
  - The plastic potential and yield surface are of the same shape.
  - The plastic potential and yield surface are associated – associated flow.
  - The plastic potential and yield surface coincide – normality condition.
  - The plastic potential is governed by frictional flow based on the dissipation of energy through friction.
- *Hardening rule:* The hardening rule describes the absolute magnitude of plastic deformation and is linked to the changing size of the yield curve. That is a description of the expansion/contraction of the yield curve with the changing stress state during yield.
- *Failure criterion:* There are many interpretations as to what constitutes failure during loading of a soil mass or specimen. In terms of the critical state framework, a critical state or ultimate failure occurs when plastic shear distortion continues indefinitely without changes in volume or effective stress, or indeed of the strength of the material – a condition of perfect plasticity or:

$$\frac{\delta p'}{\delta \varepsilon_q} = \frac{\delta q'}{\delta \varepsilon_q} = \frac{\delta \upsilon}{\delta \varepsilon_q} = 0 \qquad (12)$$

However, premature 'failure' can occur in heavily overconsolidated soils under drained loading. When these soils are sheared, the deviator stress reaches a characteristic peak strength, well before the ultimate state. Significantly, material behaviour changes from work hardening to work softening following peak



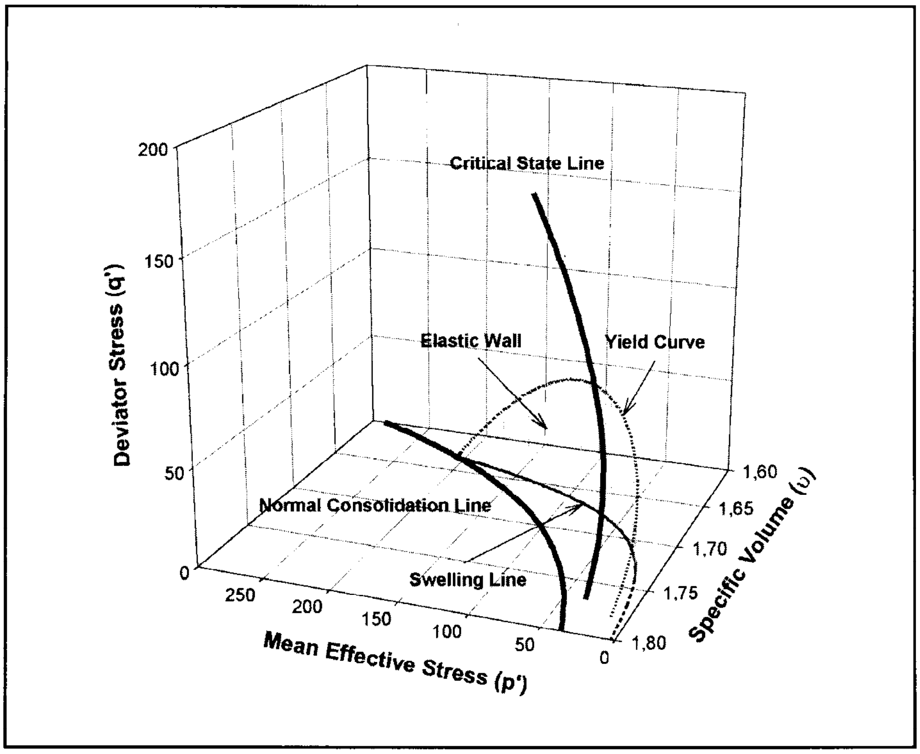


Figure 2 (a) The original Cam-Clay state boundary surface and (b) a typical yield curve and elastic wall)

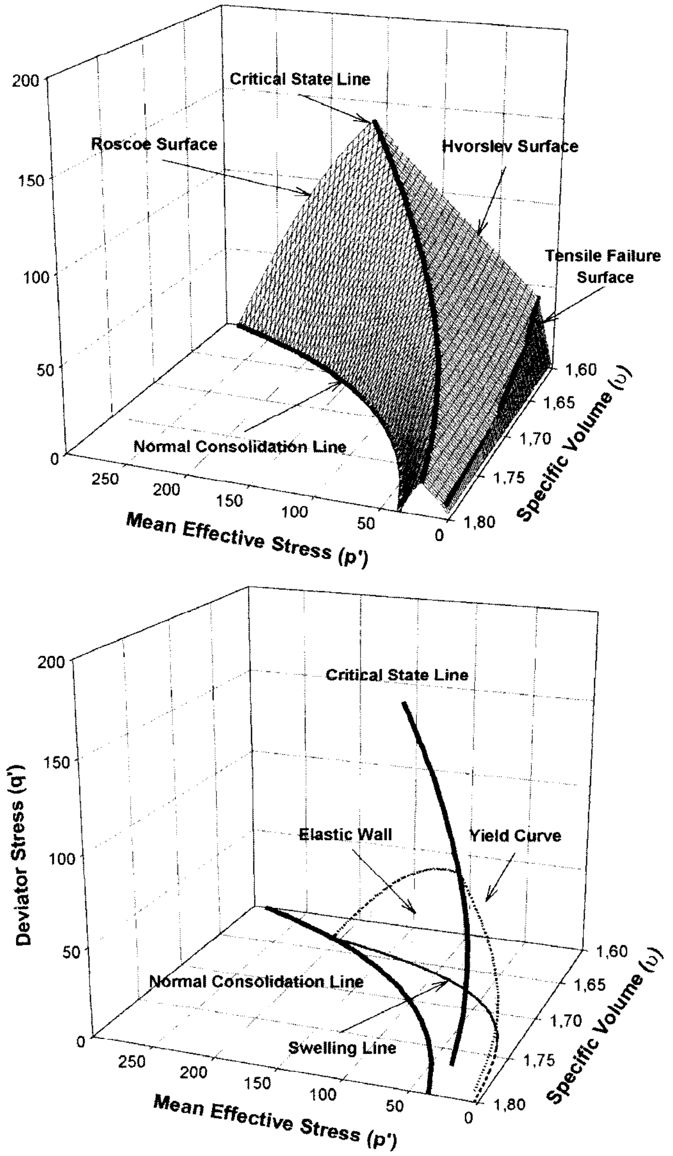


Figure 3 (a) The Schofield state boundary surface and (b) a typical yield curve and elastic wall

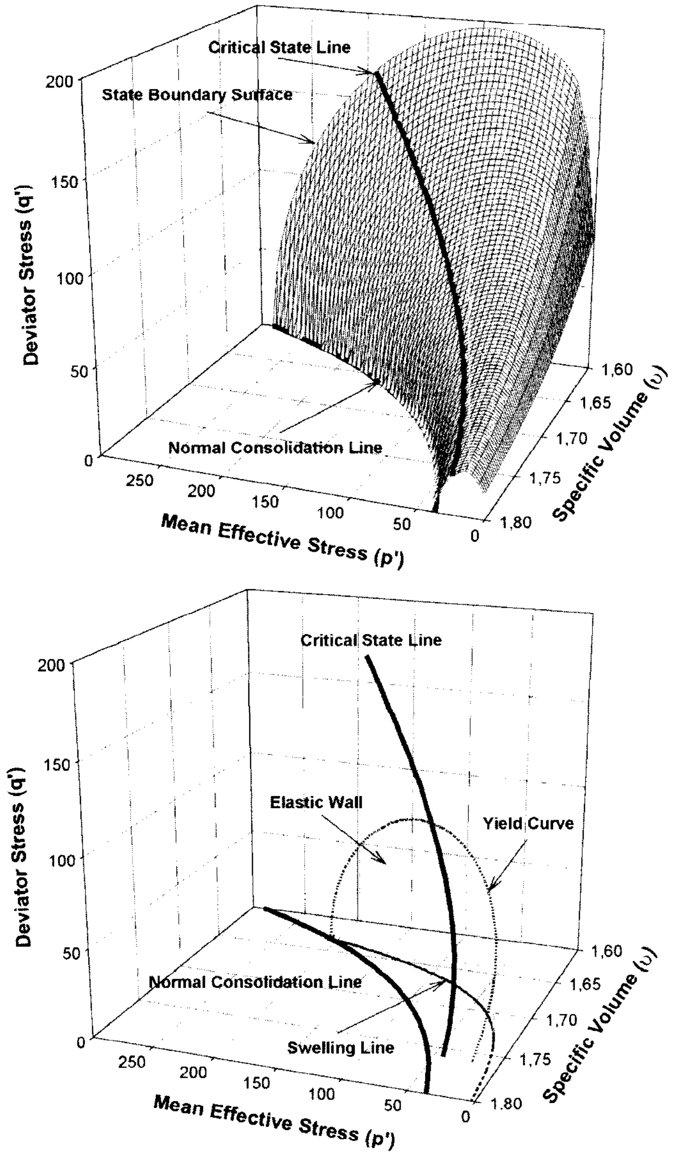


Figure 4 (a) The Modified Cam-Clay state boundary surface and (b) a typical yield curve and elastic wall)

strength. Work hardening implies that once an element, or part of the soil, has strained it becomes stronger than the rest. This element will not be subject to further straining until all other elements have strained and hardened. The result is that strains are uniformly distributed throughout the soil. Conversely, work softening implies that elements become weaker with strain, leading to a concentration of subsequent strain in the weakened zones, and thus premature failure. Strain concentration or localisation typically develops on specific planes (slip surfaces) in a soil that are associated with the orientation of the critical stresses. As it is virtually im-possible to measure localised strains on these slip surfaces, post peak triaxial strain data become unreliable.

Some definitions of critical state concepts

The following list of definitions is intended to clarify the terms used to describe critical state models which follow:

- *Stress plane:* Planer surface described by the stress invariant axes, p' and q'. Used as a convenient projection plane for illustrating the stress/strength behaviour of a soil subject to loading.
- *Compression plane:* Planer surface described by the volumetric and mean normal effective stress invariants, υ and p'. Used as a convenient projection plane for illustrating the compression behaviour of a soil subject to loading.
- *State boundary surface (SBS):* A 3D surface in invariant space (q':p':υ) that constitutes the boundary between purely elastic behaviour and elasto-plastic yield, including all possible states of ultimate failure. The SBSs for the three models used in this publication are shown in figures 2–4.
- *Normal consolidation line (NCL):* A line describing all possible states of normal consolidation, where normally consolidated refers to those states under zero deviator stress where the specimen has never been subjected to a higher mean normal effective stress in its stress history. The NCL forms part of the state boundary surface where it intersects with the compression plane and constitutes a state of yield. The line is shown in figures 2–4 and is mathematically represented by the equations of its projections onto the stress and compression planes as

$$q' = 0 \tag{13}$$

in the stress plane

$$\upsilon = N - \lambda \cdot \ln p' \tag{14}$$

in the compression plane with N and λ being soil constants

Equation 14 represents the isotropic stiffness of the material under virgin loading as a straight line in υ:lnp' with λ the slope and N the intercept at $p' = 1$ kPa.

- *Overconsolidation:* Overconsolidation describes the state of a soil which, in its stress history, has been subjected to a maximum mean normal effective stress, p'_{max}, greater than its current state. In other words, a normally consolidated soil has never experienced a mean normal effective stress greater than its current state, whereas an overconsolidated soil has been subjected to a greater mean normal effective stress in its stress history. Furthermore, a soil is described as being lightly overconsolidated or wet of critical if it yields at a density lower than its critical density (density at ultimate failure). Wet of critical implies that a saturated soil will compress and expel water under drained triaxial loading on the state boundary surface. On the other hand a soil is heavily overconsolidated if it yields at a density higher than its critical density, or dry of critical. Dry of critical implies that a saturated soil will draw in water as it dilates during drained triaxial loading on the state boundary surface. For practical purposes a heavily overconsolidated soil has an overconsolidation ratio in excess of 3 for triaxial test paths. The overconsolidation ratio can be calculated as:

$$R_p = \frac{p'_{max}}{p'} \quad (15)$$

where Rp = overconsolidation ratio in terms of p'

- *Swelling line (SL):* A line describing all possible states of overconsolidation, originating from the same pre-consoli dation stress, p'_{max}. Mathematically the SL becomes

$$q' = 0 \quad (13)$$

in the stress plane

$$\upsilon = \upsilon_\kappa - \kappa \cdot \ln p' \quad (16)$$

in the compression plane with κ being a soil constant

Equation 16 represents the isotropic stiffness of the material under overconsolidated states (unload/re-load). The parameter, κ, defines the slope in υ:lnp' and υ_κ locates the line at p' = 1 kPa. Although there exists only one unique NCL for each material, an infinite number of SLs exist, each associated with a unique pre-consolidation stress, p'_{max}.

- *Critical state line (CSL):* A line describing all possible states of ultimate failure in invariant space (q':p':υ). The critical state line is similar to the Mohr-Coulomb failure envelope and represents the ultimate shear strength of a soil at its current density or specific volume. The CSL can be expressed as:

$$q' = \text{M} \cdot p' \quad (17)$$

in the stress plane with M a soil constant

$$\upsilon = \Gamma - \lambda \cdot \ln p' \quad (18)$$

in the compression plane with Γ and λ soil constants

M represents the frictional strength of the material and is related to φ' by:

$$\text{M} = \frac{6\sin(\phi')}{3-\sin(\phi')} \quad (19)$$

or triaxial compression

Equation 18 describes the critical densities (υ) of the material at various confinement stresses. Γ locates the straight line of slope λ at a confinement of 1 kPa in υ:lnp'.

- *Yield curve:* It should be noted that purely elastic behaviour beneath the state boundary surface is confined to what are known as elastic walls. An elastic wall is a curved plane that projects from the relevant swelling line to the state boundary surface. The yield curve is formed by the intersection of an elastic wall with the state boundary surface – see figures 2–4.

One of the major advantages of critical state soil mechanics is that the parameters M, Γ, N, λ and κ are all fundamental soil constants, independent of the stress path and stress history. For this reason their values can be determined for a specific soil through tests as simple as the liquid limit and isotropic compression tests. In contrast, traditional soil parameters such as the Young's Modulus are dependent on the stress level and need to be determined by specially designed tests that simulate not only the current stress level, but also the stress history as well as the intended stress path.

For a more rigorous exposition of critical state concepts and theory, reference can be made to Roscoe and Schofield (1963), Atkinson and Bransby (1978) and Muir-Wood (1990).

CRITICAL STATE MODELS

Original Cam-Clay model (Roscoe & Schofield 1963; Atkinson & Bransby 1978)

The original Cam-Clay model was developed for the prediction of strains during an increment of elasto-plastic flow applicable to normally and lightly overconsolidated, saturated clays. The basic underlying assumptions of this model can be summarised as follows:

- Elastic recoverable behaviour is assumed ideal isotropic and follows a set of equations based on the generalised form of Hooke's Law. Isotropic elastic behaviour is decoupled in the sense that volumetric elastic strains depend only on the mean normal effective stress and shear elasticstrains only on the deviator stress, ie

$$\delta\varepsilon^e_p = \frac{1}{K'}\delta p' + 0 \cdot \delta q' \quad (20)$$

$$\delta\varepsilon^e_q = 0 \cdot \delta p' + \frac{1}{3G'}\delta q' \quad (21)$$

Poisson's ratio is assumed constant during elastic behaviour, which implies a variable shear stiffness or modulus, dependent on the current state of p' and υ (Zytynski *et al* 1978). Alternatively a constant shear modulus, G', may be assumed, which implies that the Poisson's ratio is dependent on p' and not constant. Elastic behaviour is generally non-linear, except for instances where the changes in p' and υ are relatively small, as is the case during undrained behaviour on the elastic wall. Although no elastic shear strains were originally assumed to occur in an increment of elasto-plastic flow, elastic shear strains will be calculated according to Hooke's equations for the purposes of this paper.

- The yield surface separating states of purely elastic behaviour from elasto-plastic yield consists of an infinite number of yield curves in the shape of log-spirals for the original Cam-Clay model.
- The flow rule is derived, considering the work dissipated in frictional shear after Taylor (1948), as

$$\frac{d\varepsilon^p_p}{d\varepsilon^p_q} = M - \frac{q'}{p'} \quad (22)$$

where ε^p_p = plastic volumetric strains

ε^p_q = plastic shear strains

In addition the flow rule also obeys the associated flow rule with the normality condition, essentially pinning down the shape of the yield curve and ultimately the complete state boundary surface.

- The hardening law follows from the expansion of yield curves at constant shape, the size being controlled by the pre-consolidation stress, p'_{max}, which changes with plastic volumetric strain. This expansion/contraction of the yield curve (hardening/softening) is linked with the normal compression behaviour of the soil.
- Ultimate failure occurs at a critical state defined by the critical state line and satisfies the conditions of no change in volume, effective stress and strength with continued plastic shear distortion.

The original Cam-Clay state boundary surface produced by the above assumptions simplifies to

$$q' = \frac{M \cdot p'}{\lambda - \kappa}\left[\text{N} - \upsilon - \lambda \ln(p')\right] \quad (23)$$

with $\text{N} - \Gamma = \lambda - \kappa \quad (24)$

where M, Γ or N, λ and κ are all fun damental soil constants to be derived for each soil

These constants, together with either the Poisson's ratio in terms of effective stresses (υ') or the shear modulus (G'), required for elastic calculations, result in essentially a five-parameter model. Figure 2 illustrates the Cam-Clay state boundary surface in invariant space for compressive deviator stresses.

With regard to the conditions of undrained (zero volume change) and drained ($\delta q' = 3\delta p'$) loading in a triaxial test, a set of expressions can be derived to solve for the stress paths of each. In summary these expressions are

- Purely elastic behaviour, governed by Hooke's law, becomes:

$$\begin{bmatrix} \delta\varepsilon_p^e \\ \delta\varepsilon_q^e \end{bmatrix} = \begin{bmatrix} 1/K' & 0 \\ 0 & 1/3G' \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q' \end{bmatrix} \quad (25)$$

where ε^e_p = elastic volumetric strains
ε^e_q = elastic shear strains

$$K' = \frac{\upsilon \cdot p'}{\kappa} \quad (26)$$

the bulk modulus/stiffness

$$G' = \frac{\upsilon \cdot p'}{\kappa} \frac{3(1-2\nu')}{2(1+\nu')} \quad (27)$$

the shear modulus/stiffness

These equations will apply for all subsequent models provided the behaviour is purely elastic or below the state boundary surface. In fact, the equations are also valid for the elastic strains that occur during yield on the state boundary surface.

- For a small undrained increment of elasto-plastic flow or yield in a standard triaxial compression path, elastic strains are given by equation 25 and simultaneous plastic strains by

$$\begin{bmatrix} \delta\varepsilon_p^p \\ \delta\varepsilon_q^p \end{bmatrix} = \begin{bmatrix} -\kappa/(\upsilon \cdot p') \\ -\kappa/[\upsilon \cdot p' \cdot (M - q'/p')] \end{bmatrix} \delta p' \quad (28)$$

The net volumetric and deviatoric strains can then be calculated as the sum of the elastic and plastic components during yield.

$$\varepsilon_p = \varepsilon_p^e + \varepsilon_p^p \quad \text{net volumetric strain (29)}$$

$$\varepsilon_q = \varepsilon_q^e + \varepsilon_q^p \quad \text{net deviatoric strain (30)}$$

Although the net volume change has to be zero during an undrained increment of yield, equal but opposite elastic and plastic volumetric strains are occurring at the same time.

- For a small drained increment of flow (elasto-plastic) during a standard triaxial compression path, elastic strains are given by equation 25 and simultaneous plastic strains by

$$\begin{bmatrix} \delta\varepsilon_p^p \\ \delta\varepsilon_q^p \end{bmatrix} = \frac{(\lambda - \kappa)}{M \cdot \upsilon \cdot p'} \cdot \begin{bmatrix} (M - q'/p') & 1 \\ 1 & (M - q'/p')^{-1} \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q' \end{bmatrix} \quad (31)$$

Schofield model (Roscoe & Schofield 1963; Atkinson & Bransby 1978)

The original Cam-Clay model was developed for normally consolidated and lightly overconsolidated soils. To model heavily overconsolidated soils the Cam-Clay yield curve has been extended by addition of the Hvorslev surface and a tensile failure surface on the dry side of critical. This extended form of the original model is unofficially known as the Schofield model. The original Cam-Clay yield surface on the wet side of critical, also known as the Roscoe surface, still connects the normal consolidation line to the critical state line. From the critical state line the planer Hvorslev surface extends to the tensile failure surface, which represents a boundary on the dry side of critical associated with the onset of tensile stresses. A stress path touching the tensile failure surface leads to immediate failure by separation as there is no cohesive strength by assumption. (See figure 3 for an illustration of the Schofield model.)

In the Schofield model the Roscoe surface uses exactly the same equations as the original Cam-Clay model. The relevant equations for the Hvorslev and tensile failure surfaces follow from the same principles and can be summarised as follows:

- The Hvorslev state boundary surface:

$$q' = (M - h) \cdot \exp\left(\frac{\Gamma - \upsilon}{\lambda}\right) + h \cdot p' \quad (32)$$

where h, the slope of this surface in the stress plane, constitutes an extra soil constant

The Schofield model therefore becomes a six parameter model.

- For a small increment of elasto-plastic flow in a standard triaxial compression test, elastic strains are given, as before, by equation 25. Simultaneous plastic volumetric strains reduce to

$$\delta\varepsilon_p^p = \frac{\delta p'}{\upsilon \cdot p'} \left[\lambda \left(\frac{\delta q'/\delta p' - h}{q'/p' - h} \right) - \kappa \right] \quad (33)$$

and plastic shear strains to

$$\frac{d\varepsilon_p^p}{d\varepsilon_q^p} = M - \frac{q'}{p'} \quad (22)$$

These equations, together with the relevant load conditions, either undrained ($\delta\varepsilon_p = 0$) or drained ($\delta_q' = 3\delta p'$), allow for the calculation of test paths to failure for triaxial compression tests.

- The tensile failure surface is given by

$$q' = 3p' \text{ in the stress plane} \quad (34)$$

$$\upsilon = \upsilon_{TF} - \lambda \cdot \ln p' \quad (35)$$

in the compression plane

where υ_{TF} can be calculated from the other soil constants

Modified Cam-Clay model (Roscoe & Burland 1968; Muir-Wood 1990)

It was later discovered that the shape of the original Cam-Clay yield curve, being log-spiral, did not fit experimental data satisfactorily. By changing the shape of the yield curve to that of an ellipse, much better results were found and led to the development of the Modified Cam-Clay model.

Assumptions remain essentially the same as with the original model, with the exception of the shape of the yield curve. The flow rule, which does not satisfy the frictional work dissipation criterion any longer, still holds the associated flow and normality conditions.

The shape of the elliptic yield curve is represented by the equation

$$\frac{p'}{p'_{max}} = \frac{M^2}{M^2 + \eta^2} \quad (36)$$

which leads to the equation for the Modified Cam-Clay state boundary surface (figure 4), or

$$q' = \sqrt{M^2 \cdot p' \left[EXP\left(\frac{N - \upsilon - \kappa \cdot \ln p'}{\lambda - \kappa} \right) - p' \right]} \quad (37)$$

$$\text{with} \quad N - \Gamma = (\lambda - \kappa) \ln 2 \quad (38)$$

where M, Γ or N, λ and κ are all fundamental soil constants

- For a small increment of elasto-plastic flow on the state boundary surface, elastic strains are given, as before, by equation 25, but simultaneous plastic volumetric strains by

$$\begin{bmatrix} \delta\varepsilon_p^p \\ \delta\varepsilon_q^p \end{bmatrix} = \frac{(\lambda - \kappa)}{\upsilon \cdot p' (M^2 + \eta^2)} \cdot \begin{bmatrix} (M - \eta^2) & 2\eta \\ 2\eta & 4\eta^2/(M - \eta^2) \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q' \end{bmatrix} \quad (39)$$

Equation 39, together with the load condition, undrained ($\delta\varepsilon_p = 0$) or drained ($\delta_q' = 3\delta p'$), allows for the prediction of triaxial test paths.

The Modified Cam-Clay model does not traditionally include the Hvorslev surface on the dry side of critical. However, for this paper the Hvorslev surface was subsequently added to this model for comparison with the Schofiled model.

EXPERIMENTAL DATA

The classic series of triaxial tests performed on remoulded Weald Clay at Imperial College in the 1950s by Bishop and Henkel (1962) were selected as a reference set of experimental data. Standard triaxial compression tests were performed on normally consolidated and heavily overconsolidated ($R_p = 24$) samples under both undrained and drained load conditions. The results cover a diverse range soil behaviour and provide an excellent reference for comparison with the critical state model predictions. Figures 5 to 8 on pp 8 and 9 summarise the original test results.

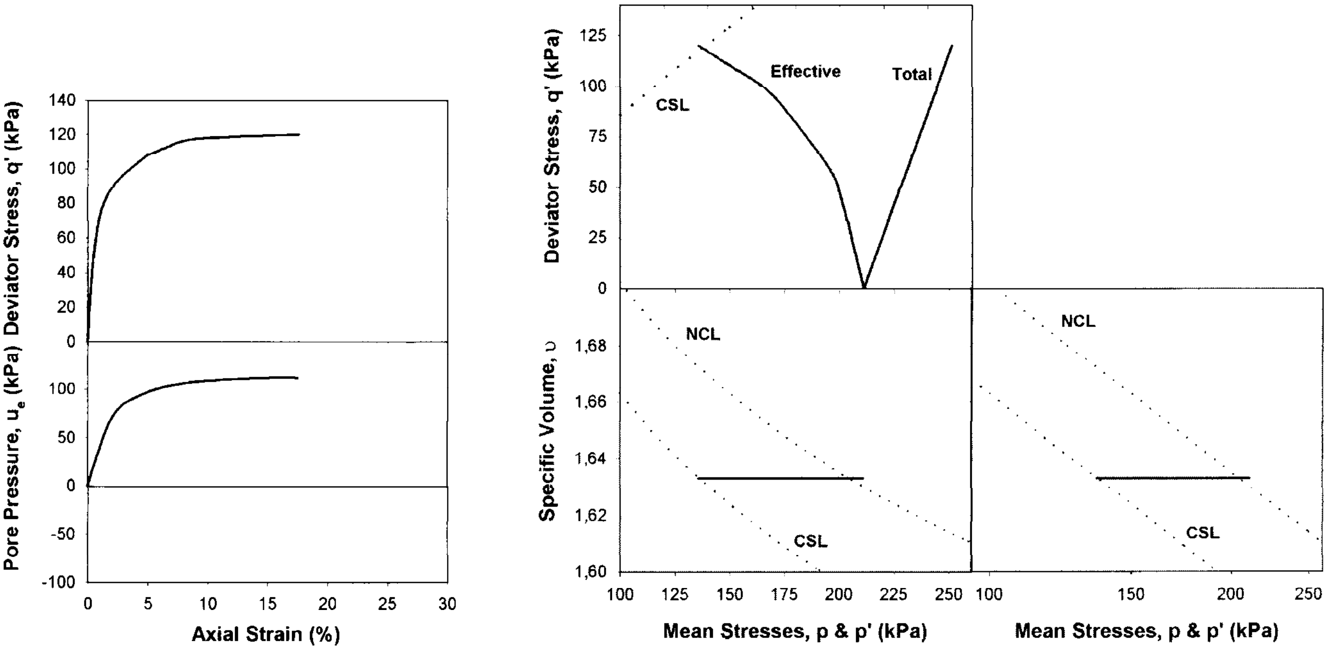


Figure 5 Undrained triaxial compression on a normally consolidated sample

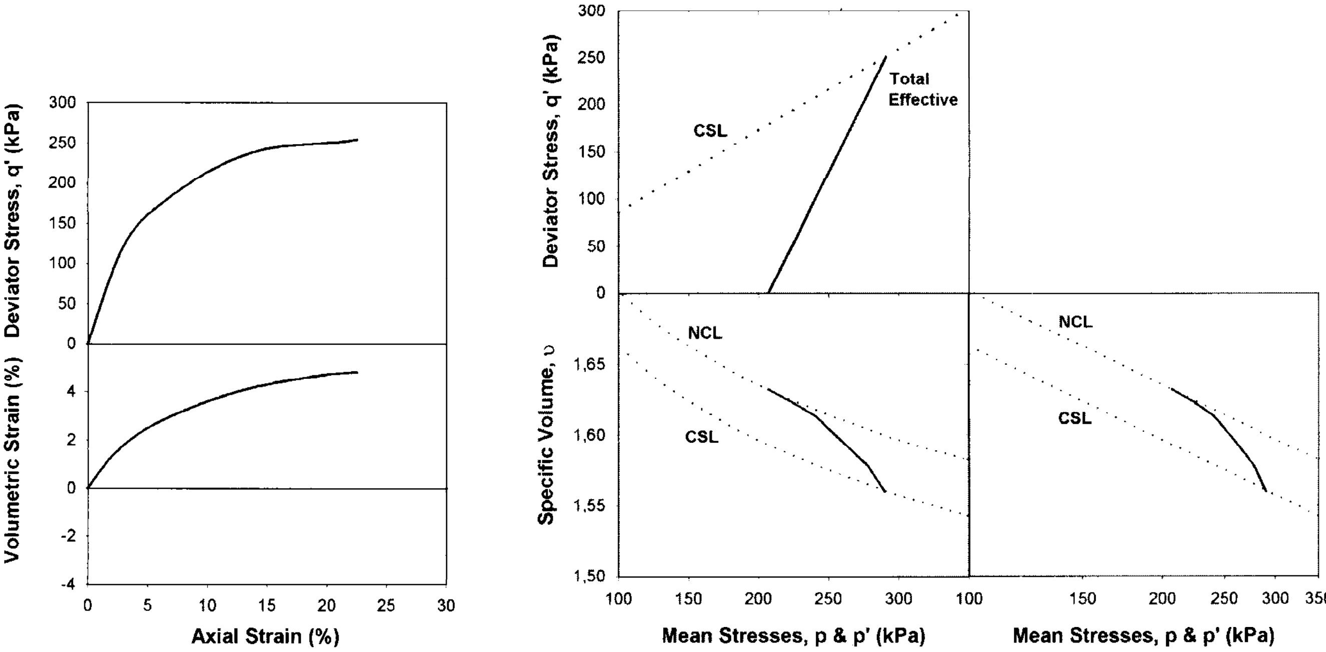


Figure 6 Drained triaxial compression on a normally consolidated sample

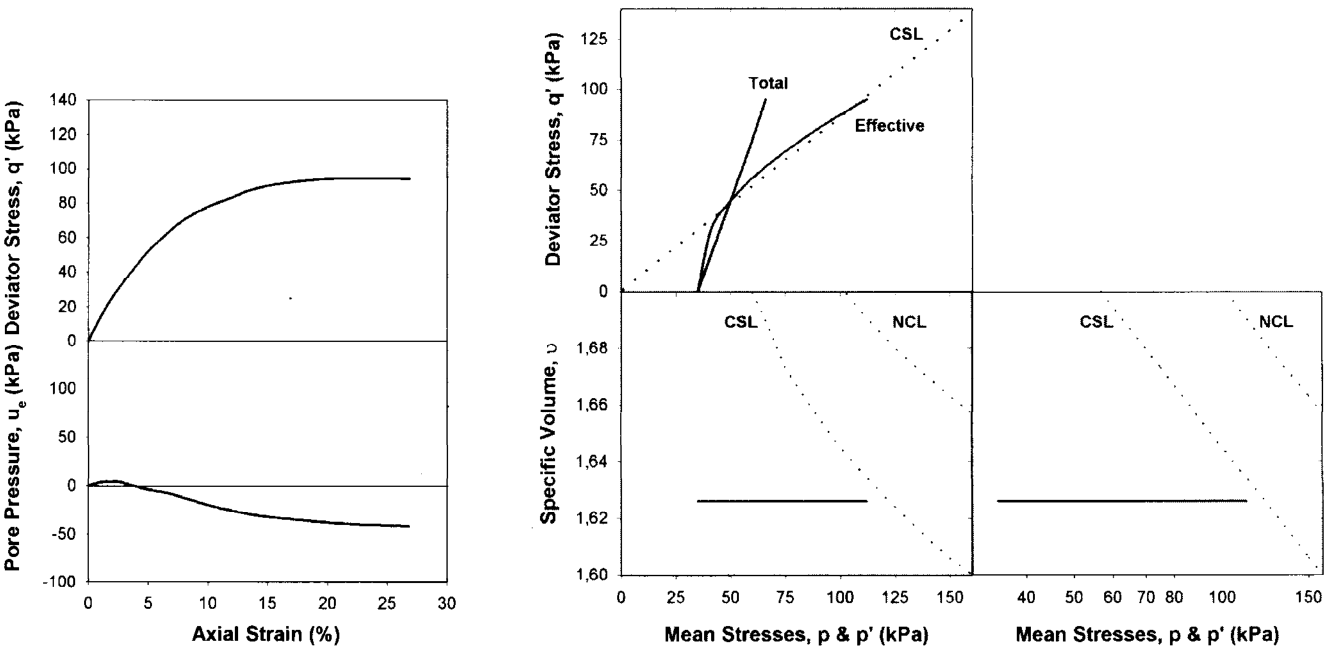


Figure 7 Undrained triaxial compression on a heavily overconsolidated sample

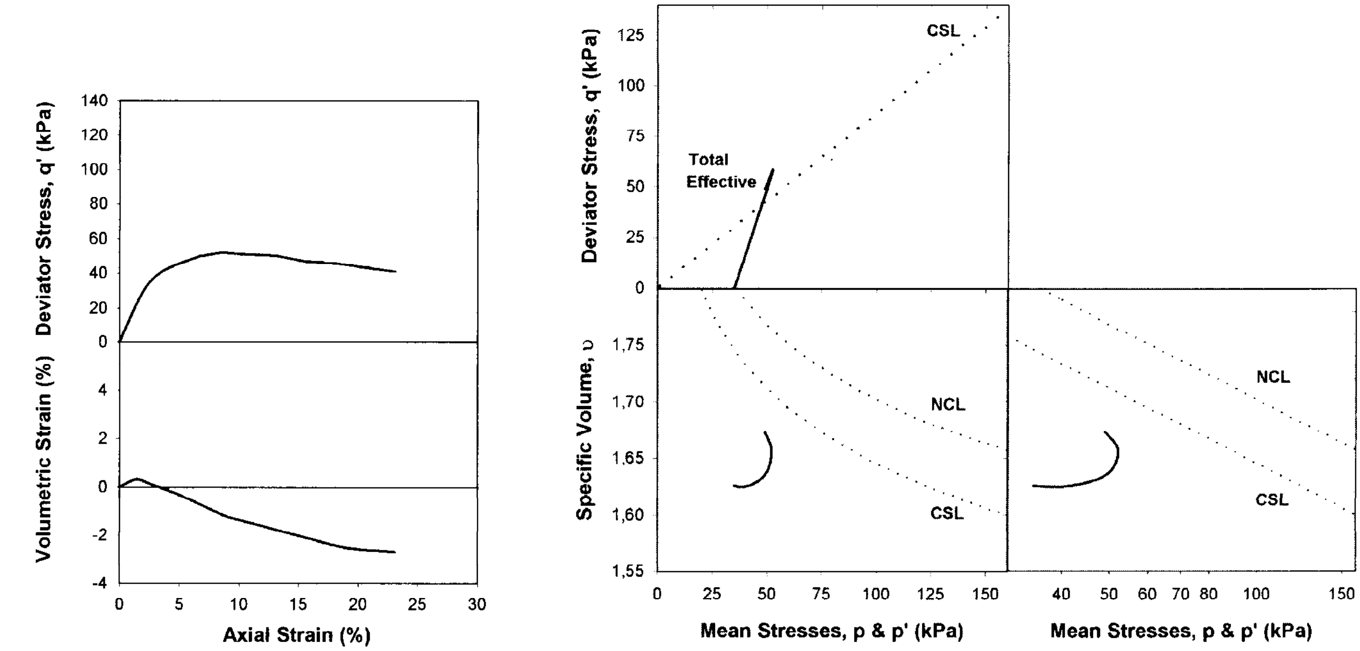


Figure 8 Drained triaxial compression on a heavily overconsolidated sample

The two normally consolidated samples were set up for triaxial shear testing by using remoulded specimens of Weald Clay and compressing them isotropically to 207 kPa. The specimens ended up with a saturated moisture content of 23%, which relates to a void ratio of e = 0,632, or specific volume of υ = 1,632. The compression stress of 207 kPa was enough to ensure that an initially remoulded specimen would start the shear phase in a normally consolidated state. Two heavily overconsolidated samples were produced by initially compressing remoulded specimens to 827 kPa and then allowing them to swell under a reduced load of 34,5 kPa, thus building in an overconsolidation ratio of R_p = 24. These specimens had a moisture content of 22,7%, void ratio of 0,626 and specific volume of 1,626. Note that there was very little difference between the void ratios of the normally and heavily overconsolidated specimens. Therefore, both the normally consolidated and overconsolidated specimens had more or less the same density, but different stress histories. This difference in stress history, as well as the mode of shear, undrained or drained, lead to the varied patterns of behaviour shown in figures 5 to 8.

During an undrained triaxial test the drainage lead connected to the specimen pore water is shut off, blocking any movement of water to or from the specimen. Water and soil grains are assumed incompressible for the range of engineering stresses usually applied and with the use of de-aired water for testing purposes. Therefore, a saturated specimen subject to undrained shear cannot change volume and is essentially a constant volume test. This fact is illustrated by the compression stress paths (υ vs p') in figures 5 and 7 and becomes a constraint when predicting test results with the critical state models. The soil responds to the condition of zero volume change by generating excess pore pressures, ue, comprising the difference between the total and effective stress paths. The total stress path q vs p is restricted by the boundary conditions of a triaxial cell to one in which $\delta q = 3\delta p$.

When a drained test is performed water is free to drain to and from the specimen voids. In fact, the strain rate of these tests is controlled by a minimum allowable excess pore pressure build-up. With the boundary conditions applied in a drained triaxial test, both the effective stress path (q' vs p') and total stress path (q vs p') are restricted to a slope of 1:3, as shown in figures 6 and 8.

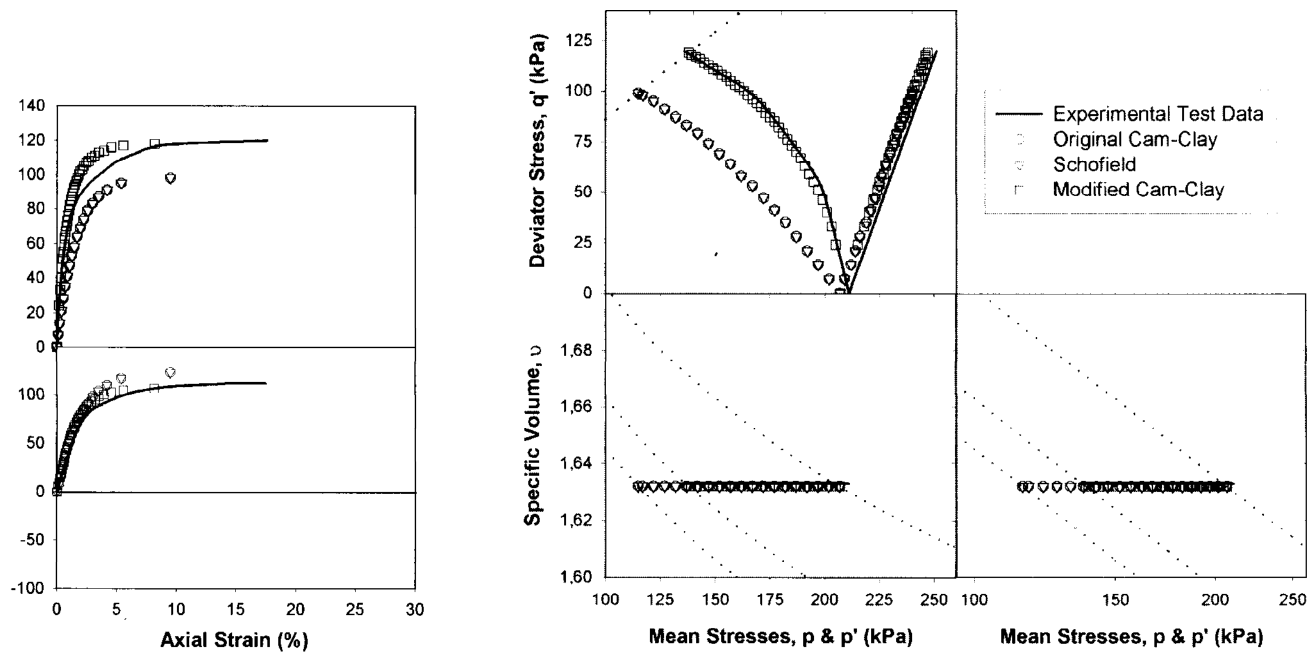


Figure 9 Undrained triaxial compression on a normally consolidated sample

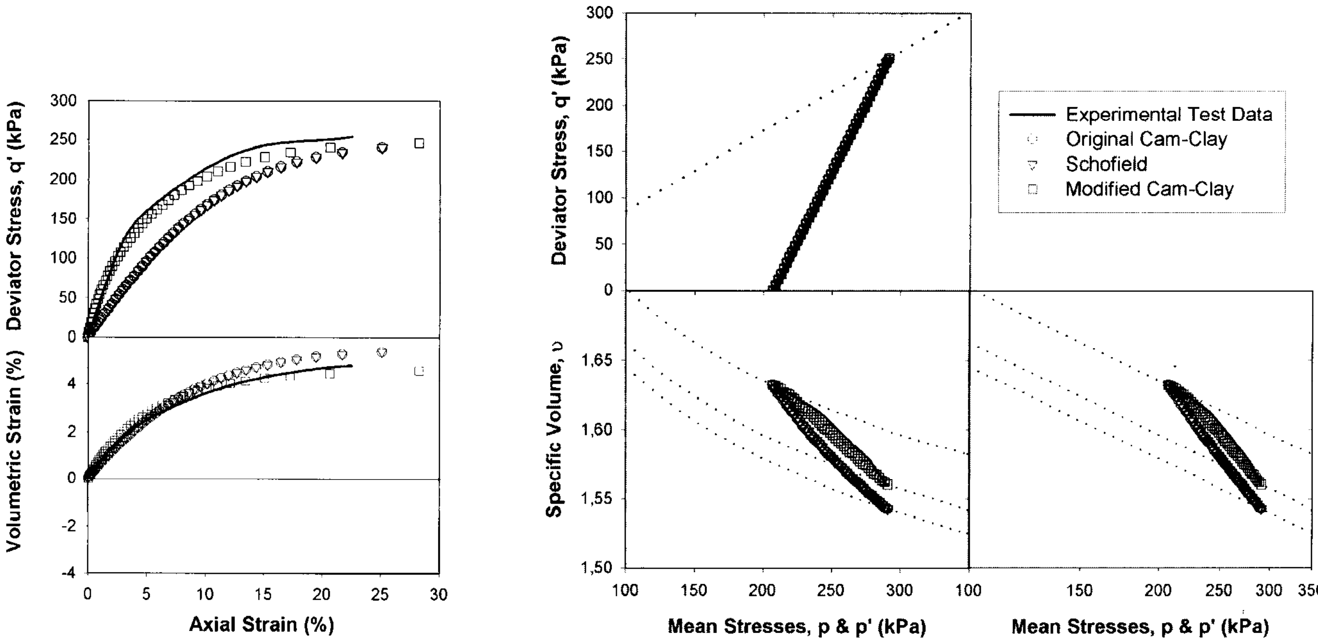


Figure 10 Drained triaxial compression on a normally consolidated sample

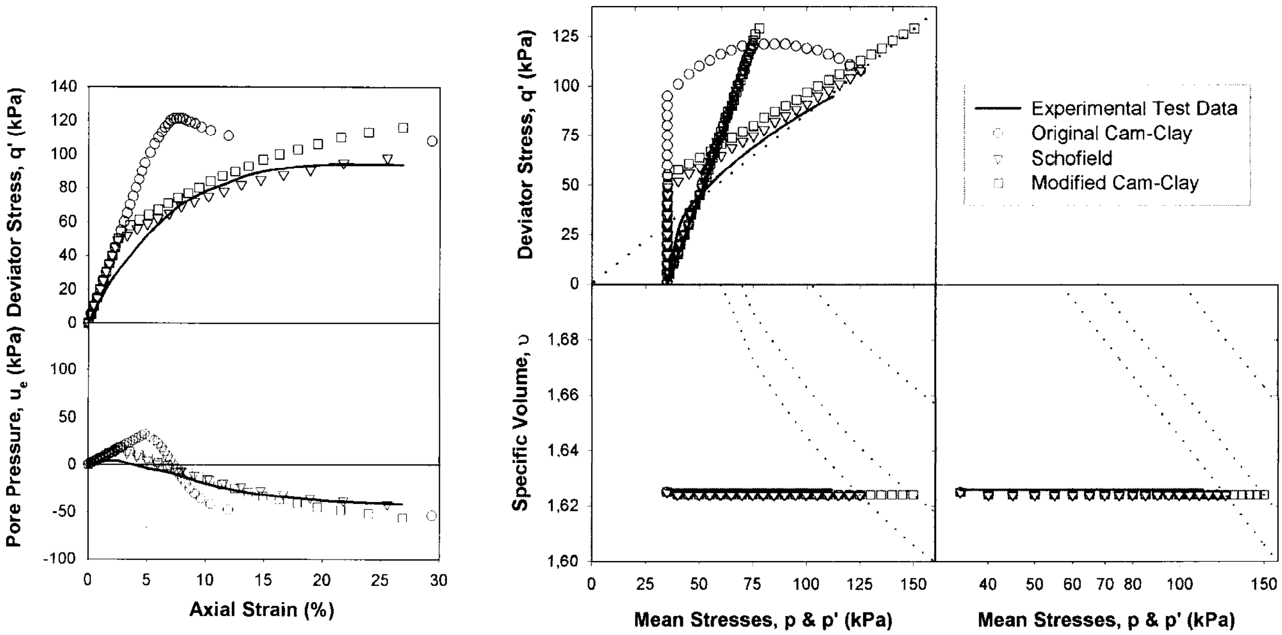


Figure 11 Undrained triaxial compression on a heavily overconsolidated sample

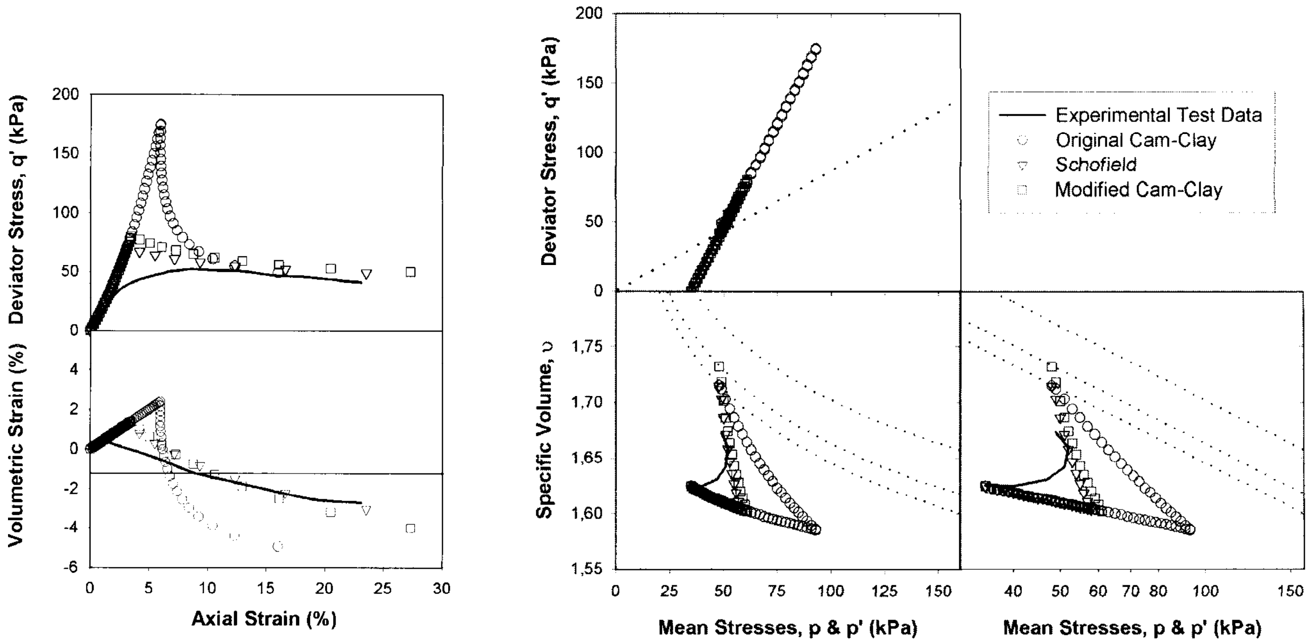


Figure 12 Drained triaxial compression on a heavily overconsolidated sample

MODEL PREDICTIONS

The results from the previous section were used in conjunction with the critical state concepts to calculate a set of fundamental soil parameters through best fit and back analysis procedures. These can be compared with similar values for the same and other clay soils from Schofield and Wroth (1968) in table 1. The two different values calculated for Γ in table 1 result from the specific assumptions of firstly, the original Cam-Clay model and secondly, the Modified Cam-Clay model.

Triaxial test paths were next generated using the critical state models and assuming a constant Poisson's ratio of $\upsilon' = 0{,}3$. The results are shown in figures 9 to 12. The compression planes show two critical state lines – these correspond to the different Γ-values of the Cam-Clay and Modified Cam-Clay models respectively.

The performance of the critical state models in predicting soil behaviour, under triaxial load conditions, can be summarised as follows:

- *Figure 9 – Normally Consolidated, Undrained:* All three models predict the pore pressure response with axial strain fairly accurately. It is clear, however, that the Modified Cam-Clay model follows the experimental stress path much closer in the stress plane. The Modified Cam-Clay model also gives a better representation of the shear stiffness of the material (q' vs ε_a) at low strain, which should be more appropriate to field loading conditions. There is no difference between the Cam-Clay and Schofield models as yielding occurs entirely on the Roscoe surface from a normally consolidated state. The Modified Cam-Clay model gives a more accurate ultimate strength but the original Cam-Clay estimate is conservative.
- *Figure 10 – Normally Consolidated, Drained:* Again the entire load path is along the Roscoe surface, which excludes any benefits to be gained from the Hvorslev state boundary surface. For the drained test on a normally consolidated specimen all three models give exactly the same strength due to the boundary conditions, but the Modified Cam-Clay model follows the test path in the compression planes much closer and gives a better representation of the material stiffness in q' vs ε_a. Volume change predictions are essentially the same and fairly accurate.
- *Figure 11 – Heavily Overconsolidated, Undrained:* The advantage of incorporating the Hvorslev state boundary surface now becomes apparent as the sample state moves dry of critical. The original Cam-Clay and Modified Cam-Clay models were never intended to model heavily overconsolidated behaviour without the inclusion of the Hvorslev SBS. There are only minor differences between the Schofield model and Modified Cam-Clay with the Hvorslev surface, both give fair representations of stiffness, strength and excess pore pressure values, with the Schofield model being slightly superior. Although the predicted stress paths indicate a sharp change in direction at the yield point, the experimental data follow a more rounded and natural path.
- *Figure 12 – Heavily Overconsolidated, Drained:* Models incorporating the Hvorslev surface are again far superior to the original Cam-Clay model. A significant feature of these test paths is the onset of strain softening following peak deviator stress. Experimental data becomes unreliable with the onset of strain localisation and the formation of planer discontinuities. For this reason the experimental test path, on the compression plane, terminates well before the critical state is reached.

Table 1 Critical state parameters for common clay soils from the UK

Soil	M	λ	κ	N	Γ	h
Weald clay (this paper)	0,863	0,096	0,04	2,144	2,088 2,105	0,650
Weald clay	0,950	0,093	0,035	2,118	2,060	
London clay	0,888	0,161	0,062	2,858	2,759	
Kaolin	1,020	0,260	0,050	3,977	3,767	
Cowden Till	1,100	0,077	0,015	1,915	1,885	

CONCLUSIONS

With the advent of the personal computer and the development of powerful numerical techniques such as finite difference and finite element methods, a number of software packages have become available to the engineer as analysis tools. The danger, as is the case with 'black-box' type solutions, is that, irrespective of input, an unqualified answer is always given.

In this paper three basic critical state constitutive models were used to predict the results of four different triaxial compression tests. It was shown that gross miscalculations can result when a constitutive model is applied without a proper understanding of its fundamental assumptions and limitations. An example would be the use of the original Cam-Clay model on heavily overconsolidated soils without the inclusion of a Hvorslev surface on the dry side of critical. On the other hand, very good predictions resulted when models were being used within their scope. Of the three models considered, Modified Cam-Clay with the Hvorslev surface on the dry side of critical, performed consistently and predicted the stress paths and failure conditions with fair accuracy.

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