# The buckling resistance of hollow circular members with flattened end-connections

**TECHNICAL PAPER**
*Journal of the South African Institution of Civil Engineering,* 44(2) 2002,
Pages 23–27, Paper 515

**N W Dekker**

***Circular hollow sections are commonly used in lattice towers. The geometry of hollow circular sections provides particularly effective compression members. Fully developed welded end-connections are generally considered to be uneconomical and flattened end connections provide economical benefits and simple fabrication. The process of flattening the ends of circular hollow sections reduces the overall stiffness of the member and introduces eccentricities at the junctions. The reduction in stiffness and the influence of such eccentricities are not taken into account in the design process. This paper examines the influence of the flattened ends and the eccentricities and recommends a rational design process to account for such effects.***

## INTRODUCTION

The design of elements subjected to axial loads is commonly performed on members with a uniform cross-section over the length of the member. Variations in the boundary conditions at the ends of the strut are compensated for by the use of different values of the effective length factor in order to provide comparison with the known strength of a pin-ended strut. No consideration is commonly given to the influence of end-connections having greatly reduced stiffness occurring over a significant length of the strut.

Circular hollow sections are commonly used in the fabrication of lattice towers. Fully developed and welded end-connections would render the use of circular sections uneconomical in structures where aesthetics are not considered to be of great importance. Flattening the ends and bolting the element directly to a gusset plate is a common method of connecting circular sections. This method obviously weakens the member by reducing the moment of inertia over a portion of the length.

Where the length of this flattened portion is small in comparison to the overall length between supports, the assumption of a prismatic strut with a uniform cross-section over the full unbraced length may be valid, but the validity of such assumptions over a range of geometric properties are not known. The flattened length becomes a significant proportion of the overall length of the member in some lattice structures where members intersect at extremely flat angles. In such cases, the buckling resistance of the member may be significantly affected.

In this paper, the influence of flattened ends is examined on both a theoretical and experimental basis.

## THEORETICAL MODEL

Solutions for the elastic buckling loads of struts with variable cross-sections have been formulated by Timoshenko and Gere (1961), Fraser and Bridge (1990), and numerical solutions by Arabi and Li (1991). These solutions essentially consider the uniqueness of the buckling load and conditions of compatibility between the portions of the strut. The approach followed in this paper is similar, but by using a principle of modal extrapolation, the behaviour of the strut is clearly demonstrated.

Consider a pin-ended strut consisting of a stiff section ($I_2$) of length 2L and two sections of length $\alpha L$ having a reduced stiffness ($I_1$). This model is particularly representative of struts with weak end-connections, having the same stiffness and length of connection at both ends. By assuming symmetry at the centre of the strut, the problem is greatly simplified.

The displacement functions for the two portions of the strut may be written as follows:

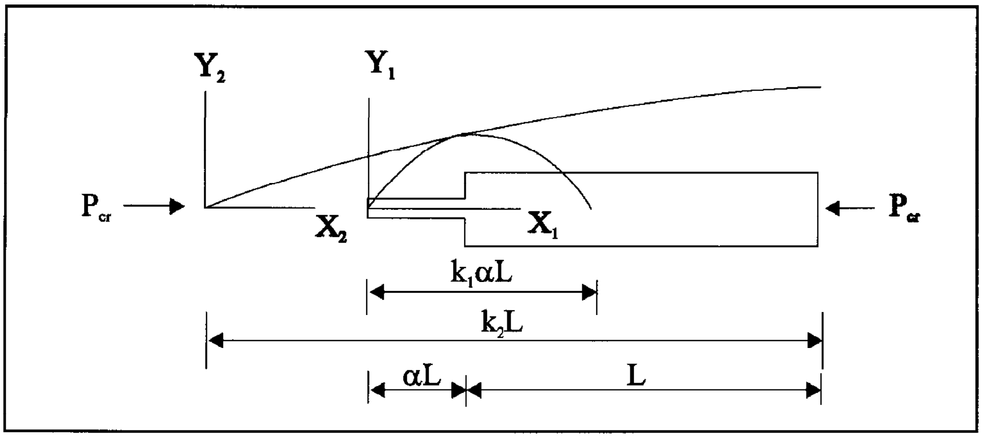

*Figure 1 Theoretical model*

NICK DEKKER received the degrees BScEng, BEng Hons and MEng from the University of Pretoria and a PhD from the University of the Witwatersrand. He spent most of his professional career with BKS where he was responsible for the design of a wide range of structures including bridges, industrial and commercial buildings, shopping centres, sports stadiums and process buildings. In 1996 he co-founded the practice Dekker & Gelderblom and was appointed as a Professor of Structural Engineering at the University of Pretoria. He applied for and was awarded an international research grading by the NRF in 1997. He is responsible for teaching structural design on an under- and postgraduate basis.

$$y_1 = A\sin\frac{\pi x_1}{k_1\alpha L}$$

and

$$y_2 = B\sin\frac{\pi x_2}{2k_2L}$$

$$\frac{dy_1}{dx_1} = \frac{A\pi}{k_1\alpha L}\cos\frac{\pi x_1}{k_1\alpha L}$$

and

$$\frac{dy_2}{dx_2} = \frac{B\pi}{2k_2L}\cos\frac{\pi x_2}{2k_2L}$$

At the junction of the portion of the member representing the end-connection and the portion assumed to possess gross section properties, the following conditions of compatibility exist:

$$y_1/_{x_1=\alpha L} = A\sin\frac{\pi}{k_1} = y_2/_{x_2=k_2L-L}$$

$$= B\sin\frac{\pi(k_2-1)}{2k_2}$$

(deflection)

$$\frac{dy_1}{dx_1}/_{x_1=\alpha L} = \frac{A\pi}{k_1\alpha L}\cos\frac{\pi}{k_1} = \frac{dy_2}{dx_2}/_{x_2=k_2L-L}$$

$$= \frac{B\pi}{2k_2}\cos\frac{\pi(k_2-1)}{2k_2}$$

(slope)

$$k_1\alpha\tan\frac{\pi}{k_1} = 2k_2\tan\frac{\pi(k_2-1)}{2k_2} \qquad (1)$$

There is only one critical load for the strut and this may be written in terms of either portion of the strut as follows:

$$P_{cr} = \frac{\pi^2EI_1}{(k_1\alpha L)^2} = \frac{\pi^2EI_2}{(2k_2L)^2}$$

From which:

$$k_1 = k_2\frac{2}{\alpha}\sqrt{\frac{I_1}{I_2}} \qquad (2)$$

Combining equations 1 and 2 results in the following transcendental equation:

$$\sqrt{\frac{I_1}{I_2}}\tan\frac{\pi\alpha}{2k_2\sqrt{I_1/I_2}} = \tan\frac{\pi(k_2-1)}{2k_2} \qquad (3)$$

Equation 3 has to be solved on a trial and error basis and provides values for the effective length factor $k_2$ for given values of $\alpha$ and $I_1/I_2$. The critical buckling load of the strut may then be written as

$$P'_{cr} = \frac{\pi^2EI_2}{(2k_2L)^2} \qquad (4)$$

The designer would commonly base the assumed resistance of the strut on a nominal resistance of a

$$P_{nom} = \frac{\pi^2EI_2}{(2(\alpha L+L))^2}$$

The reduced elastic critical elastic load may be written as a proportion of the nominal design load as follows:

$$P'_{cr} = \frac{(\alpha+1)^2}{k_2^2}P_{nom}$$

from which the reduction factor may be expressed as

$$\frac{(\alpha+1)^2}{k_2^2}$$

## APPLICATION OF THEORETICAL MODEL

As the reduction in strength of a circular section with flattened ends depends primarily on the ratios of inertia and length of the flattened ends to the circular section, these parameters warrant some discussion.

## RATIO OF MOMENTS OF INERTIA

In formulating the theoretical model it was assumed that the circular section is fully flattened and that the same material is deformed into two plates.

The moment of inertia of the circular section with a diameter D and a wall thickness of t, is given by

$$I_c = \frac{\pi(D-t)^3t}{8}$$

The moment of inertia of the flattened portion may be approximated by

$$I_{cf} = 1{,}047Dt^3$$

The ratio $I_1/I_2$ may therefore be expressed as

$$\frac{I_1}{I_2} = 2{,}666\left(\frac{t}{D}\right)^2$$

It may be argued that the assumption that the circular section is completely flattened results in extreme and perhaps overly conservative values of the stiffness ratio, especially when the transition piece is not accounted for. It should, however, be considered that the flattened section is particularly sensitive to bending moments induced by eccentricities.
The choice of a conservative ratio of $I_1/I_2$ was chosen in order to compensate in part for the influence of eccentricities, the magnitude of which is unknown.

For a 89 x 3 CHS the value of $I_1/I_2$ is equal to 0,004 and for a 76 x 3 CHS the value of $I_1/I_2$ is equal to 0,006. Taking these in combination with a range of values for the flattened end ratio or section length ratio, $\alpha$, the influence of the flattened ends on the elastic buckling load may be demonstrated as shown in figures 2a and 2b.

The most significant parameter influencing the buckling strength of circular members connected with flattened ends is clearly the proportion of length of the flattened end to the total length of the strut. For the two sections investigated, a flattened length of 5% of the nominal length indicates a reduction in the buckling capacity of some 20%, while increasing the flattened length to 10% reduces the buckling capacity by some 50%. This reduction in buckling strength caused by an increase in the flattened length is clearly illustrated by the rapid increase in the effective length factor.

## INELASTIC BEHAVIOUR

To account for inelastic buckling due to residual stresses and geometric imperfections, the theoretical model was merely used to determine the appropriate buckling length, adjusted for the non-prismatic properties of the strut. The equivalent slenderness ratio so obtained was then used in conjunction with the inelastic buckling curve of SABS 0162 to calculate the inelastic buckling moment. The results obtained from the first and second stages of experimental work are compared with the SABS 0162 design curve and this comparison is shown in figure 4. Note that the ULS values of SABS 0162 have

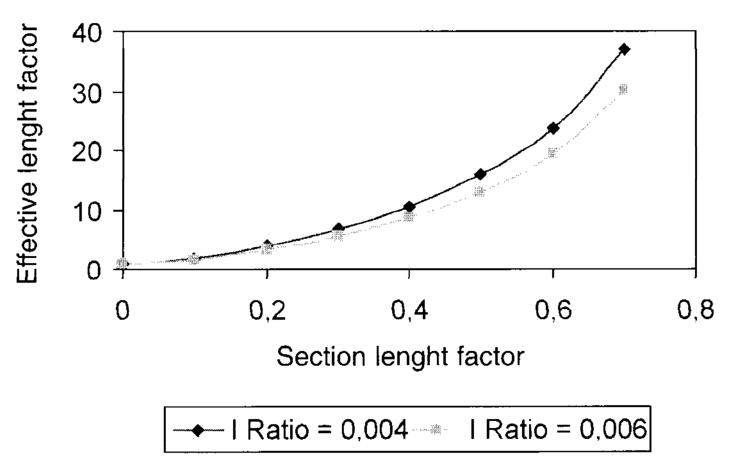

***Figure 2a Variation of effective length factor of nominal section with section length factor***

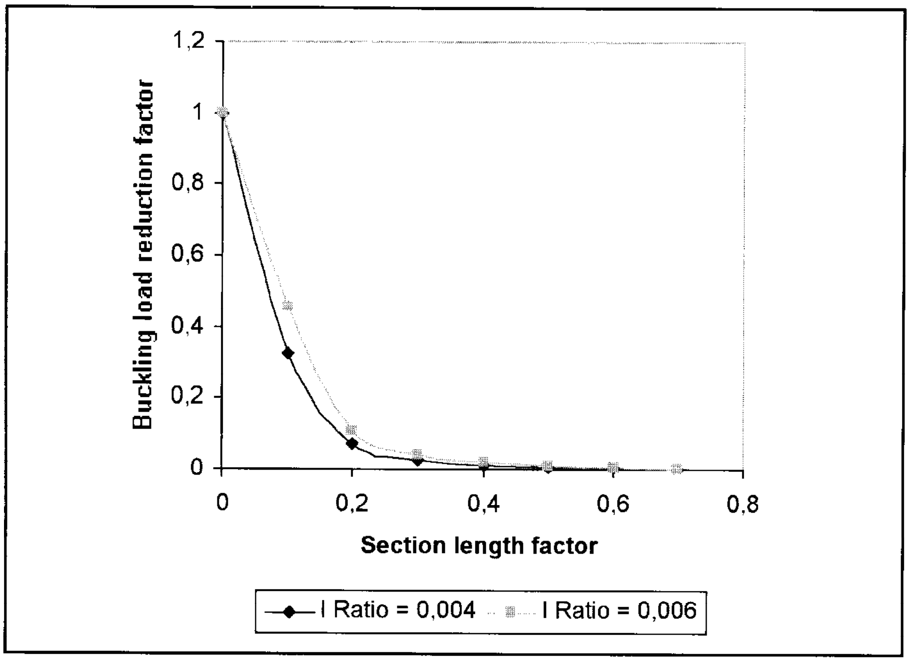

*Figure 2b Reduction in elastic buckling load for different section length factors*

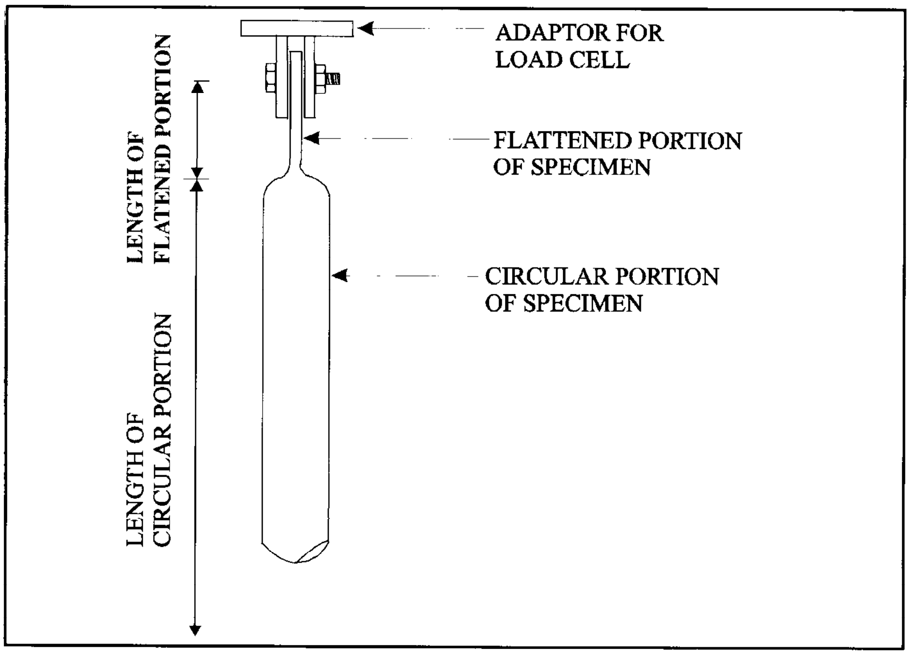

*Figure 3 Experimental setup using bolted end-connection*

been adjusted to reflect nominal strength values by dividing by a partial material factor of 0,9, thereby allowing direct comparison with experimentally obtained failure loads.

Coupons taken from some of the specimens indicate that the actual yield strength is in the vicinity of 450 Mpa. As this study was primarily aimed at investigating the influence of flattened ends at higher slenderness ratios, the actual yield strength does not play a fundamental role.

## EXPERIMENTAL WORK

Two series of experimental investigations were undertaken. In order to obtain lower bound values of the buckling loads, the experimental setup was detailed in such a manner as to minimise rotational restraint at the ends of the specimens. This approach resulted in some instability at the ends of the specimens, especially in the case of the stage 1 tests where the single bolt connection was used. Changing to the V-shaped end-connection greatly improved the situation but, even so, some rotation of the end block and the load cell was observed. It is concluded that the buckling loads measured in all the tests reflect lower bound values, but especially in the case of the bolted connections.

### Stage 1

The first stage of the experimental investigation consisted of a series of tests on specimens with flattened ends utilising bolted end-connections, as shown in figure 3.

All specimens were tested in an upright position, so eliminating bending caused by the self weight of the member. This initial investigation confirmed the sensitivity of the experimental setup with regards to eccentricities.

The experimental setup used in the stage 1 tests was not stable enough to provide reliable test results. In some cases the initial eccentricities caused the end-connection and load cell to rotate, which resulted in flexural failures at relatively low loads. This behaviour is shown in figure 4.

The results of the stage 1 tests are indicative of the sensitivity of the setup to eccentricities. In real applications, some form of fixity would always be generated by the bolted connection, reducing the influence of eccentricities in the connection itself. In most of these tests the capacity of the specimen was limited by the flexural capacity of the flattened portion at the start of the circular section.

### Stage 2

A second series of tests were conducted, but using V-shaped end pieces, thereby reducing the eccentricities to a level consistent with that probably encountered in practice. By stabilising the end-connection, the rapid formation of a mechanism was avoided. The specimens were not tested in a vertical position but were tested horizontally, with the stems of the flattened portions orientated in a vertical position. The influence of bending caused by the self weight of the member was therefore eliminated about the buckling axis. The experimental layout is shown diagrammatically in figure 5.

Using the V-shaped end pieces allowed tests to be carried out at higher slenderness ratios than was possible in the case of the bolted end-connection.

## SUMMARY OF PARAMETERS OF TEST SPECIMENS (TABLE 1)

C - *Control case,* b - bolted end-connection, v - v-shaped end-connection, S - test specimen with flattened ends, /1 - specimen number, - percentage flattened

## DISCUSSION OF TEST RESULTS

It was generally observed that the attempt to remove all rotational restraint and so obtain 'real' pinned connections at the ends of the strut, led to some instability of the end-connections induced by rotation of the load cell. As a result, even the tests on the control cases resulted in values of buckling loads that were lower than predicted. In the case of the bolted end-connections, the control case only achieved a load equal to 69% of the predicted load, while the two control cases tested using V-supports achieved between 77% and 79% of the predicted load.

*Table 1 Parameters of test specimens*

| Specimen reference | Section size | Nominal slenderness ratio ( L/r ) | Effective slenderness ratio ( KL/r ) | Ultimate load (predicted) | Ultimate load (measured) |
|---|---|---|---|---|---|
| Cb/1 | 76x3 CHS | 52 | 52 | 190 kN | 131 kN* |
| Sb/2-10 | 76x3 CHS | 52 | 139 | 67 kN | 5,4 kN* |
| Sb/3-10 | 76x3 CHS | 110 | 293 | 16 kN | 6,4 kN* |
| Sb/4-20 | 76x3 CHS | 52 | 273 | 19 kN | 7,4 kN* |
| Sb/5-20 | 76x3 CHS | 110 | 576 | 4,2 kN | 4,3 kN* |
| Sb/6-10 | 89x3 CHS | 43 | 138 | 79 kN | 10,1 kN* |
| Sb/7-10 | 89x3 CHS | 92 | 297 | 19 kN | 7,5 kN* |
| Sb/8-20 | 89x3 CHS | 43 | 273 | 22 kN | 10,3 kN* |
| Sb/9-20 | 89x3 CHS | 92 | 587 | 4,8 kN | 4,6 kN* |
| *Cv/1* | 89x3 CHS | 99 | 99 | 143 kN | 113 kN |
| Sv/2-10 | 89x3 CHS | 99 | 168 | 55 kN | 24 kN |
| Sv/3-5 | 89x3 CHS | 99 | 95 | 153 kN | 80 kN |
| Sv/4-10 | 89x3 CHS | 99 | 152 | 68 kN | 33 kN |
| *Cv/2* | 76x3 CHS | 105 | 105 | 109 kN | 84 kN |
| Sv/5-10 | 76x3 CHS | 105 | 158 | 53 kN | 32 kN |
| Sv/6-5 | 76x3 CHS | 105 | 109 | 102 kN | 69 kN |
| Sv/7-5 | 76x3 CHS | 105 | 120 | 86 kN | 50 kN |

* Tests terminated by instability of specimen.

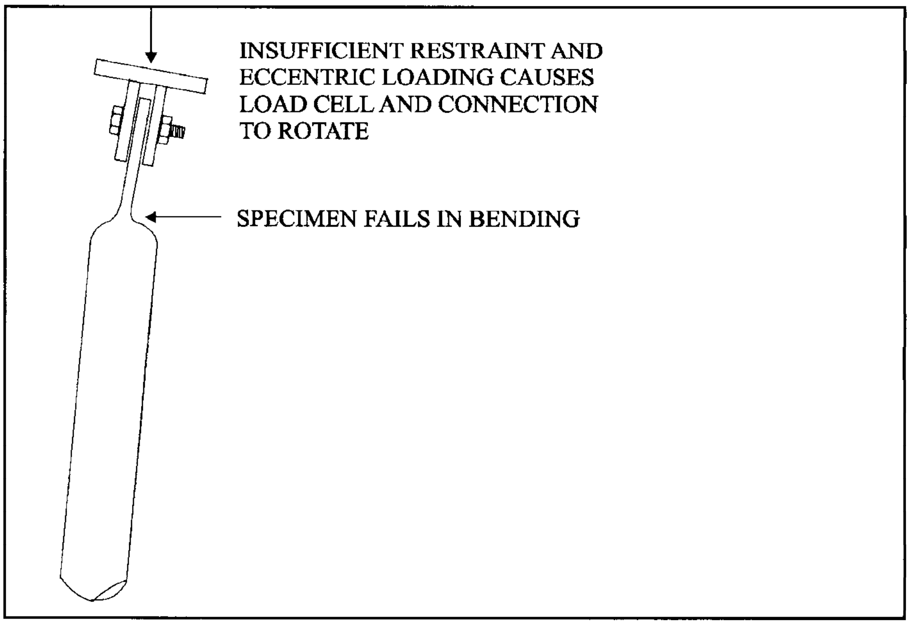

*Figure 4 Typical failure mode of stage 1 specimens*

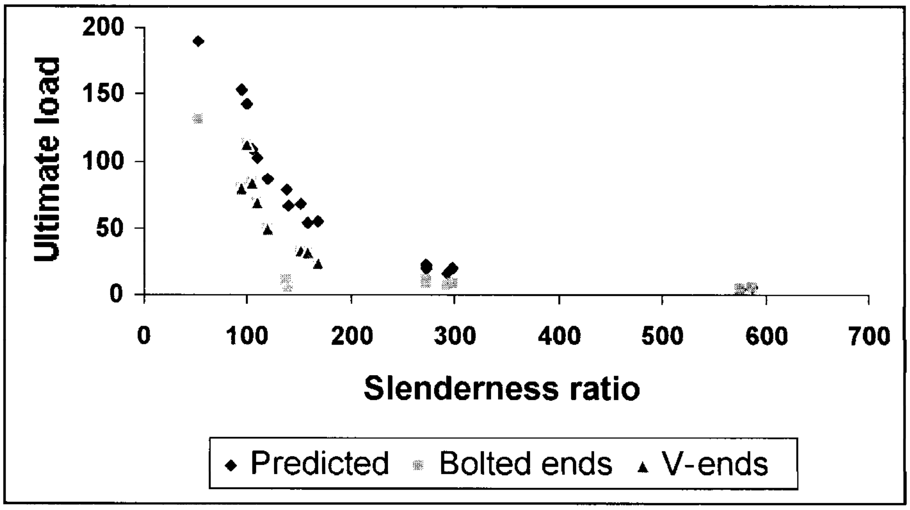

*Figure 5 Comparison of experimental results with SABS 0162 predicted values*

## EFFECTIVE SLENDERNESS RATIO METHOD

The method of adjusting the nominal slenderness ratio with an effective length factor K to account for the reduction in overall stiffness caused by the flattened ends provides an acceptable form of modelling the behaviour of such non-prismatic struts.

The discontinuities in the predicted values are due to the fact that two different section sizes were used in the series of tests. It is also clear that eccentricities have a significant influence on the strength of such struts. In the case of the specimens with the bolted end-connections, the results are perhaps a little misleading as a mechanism can form at loads that are well below the theoretical values, as the axial capacity of such struts is determined by the flexural capacity of the flattened portion. The same specimens tested using the V-shaped supports indicate that the theoretical model is capable of modelling the general behaviour, although the predicted results are always higher than the experimental values.

## INFLUENCE OF ECCENTRICITIES

The proposed effective slenderness ratio method does not account for eccentricities of loading caused by the centre line of the flattened portion not coinciding with the centre line of the circular portion (see fig 7).

Quantifying the influence of eccentricities in a non-prismatic strut on a theoretical basis introduces a level of complexity that may not be warranted given the level of uncertainty and probable conservatism surrounding the boundary conditions of the actual connection.

A simpler approach would be to increase the effective slenderness ratio by a fixed percentage in order to allow for the influence of eccentricities.

Different values of adjustment to the effective length factor were investigated in order to achieve an acceptable basis for design purposes, as reflected in table 2.

The buckling loads measured on eight test specimens are compared to theoretical values obtained using effective slenderness ratios allowing for the influence of the flattened ends. It is significant to note that even in the control cases (*Cv/1* and *Cv/2*), the buckling loads were some 20% lower than predicted values. Four different adjustment factors, 1,0, 1,2, 1,25 and 1,45, were used in table 2 to compensate for eccentricities at the flattened end junction. By increasing the effective length by a factor of 1,45, generally safe predictions may be obtained for design purposes. If, however, the test results are normalised with respect to the control cases, that is, the loads obtained from the test results are increased proportional to the ratio of load

*Table 2 Comparison of test and predicted values*

| Specimen reference | Section size | Ultimate load (measured) | Effective slenderness ratio ( KL/r ) | Ultimate load (predicted) 1,0 KL/r | Ultimate load predicted with 1,2 KL/r | Ultimate load predicted with 1,25 KL/r | Ultimate load predicted with 1,45 KL/r |
|---|---|---|---|---|---|---|---|
| Cv/1 | 89x3 CHS | 113 kN | 99 | 143 kN | 103 | 96 | 84 |
| Sv/2-10 | 89x3 CHS | 24 kN | 168 | 55 kN | 40 | 36 | 25 |
| Sv/3-5 | 89x3 CHS | 80 kN | 95 | 153 kN | 111 | 103 | 91 |
| Sv/4-10 | 89x3 CHS | 33 kN | 152 | 68 kN | 48 | 44 | 34 |
| Cv/2 | 76x3 CHS | 84 kN | 105 | 109 kN | 79 | 74 | 57 |
| Sv/5-10 | 76x3 CHS | 32 kN | 158 | 53 kN | 37 | 34 | 27 |
| Sv/6-5 | 76x3 CHS | 69 kN | 109 | 102 kN | 74 | 69 | 53 |
| Sv/7-5 | 76x3 CHS | 50 kN | 120 | 86 kN | 63 | 59 | 44 |

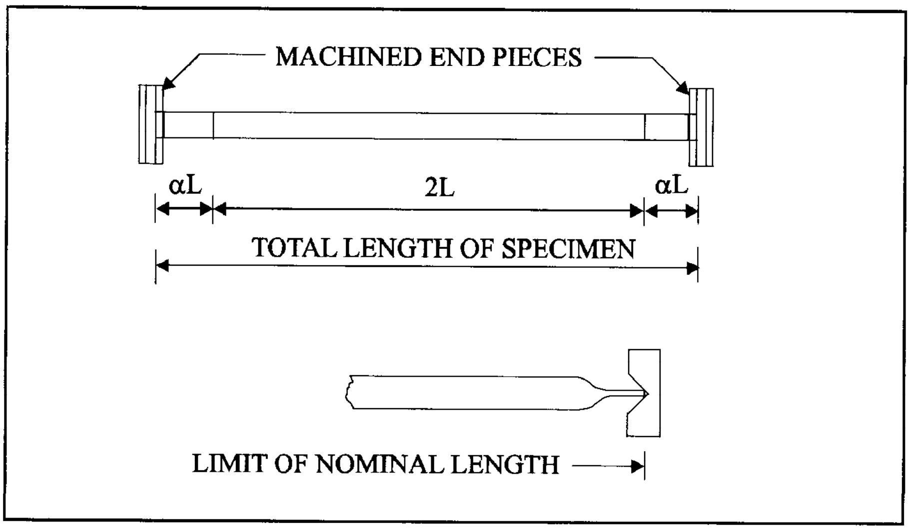

*Figure 6 Diagrammatical layout of stage 2 experimental setup*

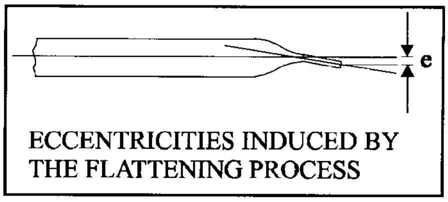

*Figure 7 Influence of eccentricities on the buckling resistance of circular sections with flattened ends*

achieved in the tests on the control cases (no flattening of ends), to the predicted load, then an adjustment factor of 1,0 would suffice. Tests on bolted specimens were not used as eccentricities caused premature failure at the stem of the flattened portion.

In 'real' structures some form of restraint would always be present in a bolted connection, and the assumption of no fixity over the nominal length of the strut should provide an additional reserve of strength in the strut.

## CONCLUSIONS

Flattened end-connections have a significant influence on the buckling strength of circular hollow sections. Where the length of the flattened portion exceeds 10% of the overall length of the member, the buckling resistance is significantly reduced. The influence of flattening the ends of a compression member is twofold: the flattening reduces the overall stiffness of the member and introduces severe local imperfections at the transition point between the circular portion and the flattened portion. The reduction in stiffness caused by the flattened portion may be quantified by applying a variable effective length factor to the member. This effective length factor is a function of the geometrical properties of the member, considering both the ratios of the moment of inertia of the flattened portion to the circular section and the length of the flattened portion to the overall length of the member. This factor cannot account for the influence of eccentricities introduced by the flattening process at the transition between the flattened portion and the circular section. Increasing the effective length by a further 45% leads to generally safe predictions, especially when considering that some form of fixity will always be present in real structures.

### Acknowledgements

The valuable assistance of the RIH group in supplying the samples, as well as the financial support of the Southern African Institute of Steel Construction, is gratefully acknowledged.

### References

Arbabi, F & Li, F 1991. Buckling of variable cross-section columns: integral-equation approach. *Journal of Structural Engineering*, 117(8):2426–2441.

Fraser, D J & Bridge, R Q 1990. Buckling of stepped crane columns. *Journal of Constructional Steel Research*, 16(1):23–38.

Gere, J M & Timoshenko, S P 1991. *Mechanics of materials*. Third edition. London: Chapman & Hall.