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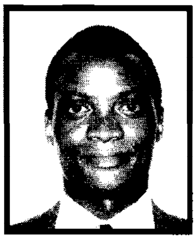
Joint type and the stability behaviour of steel frame beams

A Masarira



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The influence of the beam-to-column joints on the lateral-torsional behaviour of steel frames is investigated. Commonly used connections with various stiffening arrangements are analysed in order to determine their effect on the stability behaviour of the whole structure. Through finite-element modelling of portal frames, it is established that a consistent relationship exists between the critical load for lateral-torsional buckling of the frame beams and the joint design. A comparison between the critical loads obtained from the finite-element method, and those derived from the equivalent-member method, leads to a derivation of reliable coefficients that simulate joint-type behaviour and facilitate a simplified and effective analysis of frame beams.

INTRODUCTION

The beam-to-column joints in steel frames, which are treated as non-dimensional points in most of the conventional structural analysis methods, are in reality three-dimensional components of structures. These traditional models tend to be simplified representations of the joint characteristics and behaviour. Tschemmernergg (1957) developed a model, widely adopted in Switzerland and Austria, which is based on a relationship between bending moment and rotation represented by a rotational spring with elastic-plastic characteristics. The wind-connection method of Disque (1975) approximates joint behaviour through a bilinear moment-rotation relationship, while Eurocode 3 (1993) depicts the moment-rotation characteristic through a relationship between the moment limits, rotational stiffness and rotational capacity of the joint.

Although these and other similar models are relatively accurate in approximating the rotational stiffness of beam-to-column connections, they simplify the warping stiffness and warping torsional effects of member connections. Whereas the kinematics of beams with full cross-sections can be described in terms of the translation and rotation of each cross-section, a more accurate description of the effects of joints on the behaviour of steel frames requires the inclusion of warping stiffness and continuity conditions. An additional degree of freedom describing cross-sectional warping is necessary for torsional problems of thin-walled beams with open sections.

In this paper, a variety of joints are investigated using the Finite Element method, in order to determine their warping stiffness. Since this investigation is in the form of a comparative study to determine the relative stiffnesses of the joints, the manner in which the various plates that constitute the whole joint are connected (ie welded or bolted) was therefore deemed to be of no great significance as long as this was consistent for all joints. Whole portal frames are analysed to establish the relationship between joint design and the lateral-torsional buckling load of frames.

STABILITY DESIGN FOR FRAME BEAMS

The German Standards (DIN 18800/2, DIN 4114)

The standards of most countries adopt approaches that rarely take into consideration the nature and magnitude of the warping stiffness of the joints. However, in the German stability standards, DIN 18800/P2 (1990) and DIN 4114 (1952), two separate coefficients (β_0, β_z) are applied to take into account the effects of the degree of both the warping and bending restraints of the member ends. Since the author is familiar with the German DIN codes whose approach approximates the joint effects more realistically, these will therefore be used in this comparative study. According to the equivalent-member method (EMM), the frame analysis is conducted on the basis of isolated members whose end conditions (eg bending and warping stiffness) are approximated. According to DIN 18800/P2 (1990) the derivation of the ideal lateral-torsional buckling moment for a beam is conducted on the basis of equation (1) below, and the check for stability takes into account the boundary conditions.

$$M_{cr,y} = \xi \, N_{cr,z} \left(\sqrt{c^2 + 0{,}25 z_p^2} + 0{,}5 z_p \right) \quad (1)$$

where

$$c^2 = \frac{I_\omega \dfrac{(\beta_z l)^2}{(\beta_0 l_0)^2} + 0{,}039 (\beta_z l)^2 I_T}{I_z} \quad (2)$$

$$N_{cr,z} = \pi^2 \, . \, E \, . \, I_z \, / \, l^2 \quad (3)$$

and $N_{cr,z}$ is the axial buckling load, z_p is the distance of load application point from the shear centre, l is the beam length for bending, l_0 is the beam length for warping, I_ω is the warping moment of inertia, ξ is the moment coefficient, I_T is the Saint Venant torsional constant and I_z is the moment of inertia about the z axis (refer to fig 1).

The moment coefficients (ξ) used are for the moment distributions due to uniformly distributed load in single-span beams. The

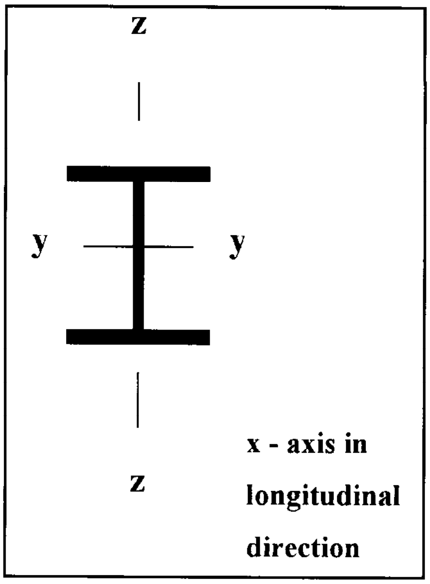


Figure 1. Reference axes for an I-beam

coefficients (β) for member end conditions vary within the ranges: $\beta_z = 1,0$ (pinned support) to $\beta_z = 0,5$ (rigid support); $\beta_0 = 1,0$ (free to warp) to $\beta_0 = 0,5$ (full warping restraint). When $\beta_z = \beta_0 = 1,0$ the beam is free to warp at its (pinned) ends, whereas the values $\beta_z = \beta_0 = 0,5$ imply full warping and rotational restraints. A modification of equation (1) leads to:

$$M_{cr,0} = \xi_0 \; N_{cr,z} \; (\xi_0 \; 0,4053 \; z_p + \sqrt{(\xi_0 .0,4053.z_p)^2 + c^2}) = q_{cr} \, . \, l^2 / 8 \qquad (4)$$

$$\text{Max } M_{cr,Span} = M_{cr,0} \, . \, \text{Max } M_{Span} / M_0 \text{ or } M_{cr,B} = M_{cr,0} \, . \, M_B / M_0 \qquad (5)$$

Max M_{Span} is the maximum bending moment in the beam span.

$$M_0 = q_z \, . \, l^2 / 8 \qquad (6)$$

The coefficient ξ_0 is dependent on the ratio ψ (where $\psi = M_B / M_0$), qz is the applied distributed load in z-direction and M_B is the moment at the beam-end due to q_z.

The check for lateral-torsional buckling is conducted on the basis of $M_{cr,Span}$. Kindmann (1993) has done some extensive work that takes into account the bending moment distribution for lateral-torsional buckling of continuous beams, and calculations of the moment coefficients ξ_0 for various moment proportions have been conducted. Using these coefficients, the critical loads q_{cr} for a frame beam for all possible combinations of β_0 and β_z can be calculated using equation (4), and such results are shown in table 1. A comparison can be made between these q_{cr} values and the critical lateral-torsional buckling loads for the same frame with different joint types, as determined using the finite-element model. In this way, it is possible to assign specific warping and bending restraint coefficients (β_0, β_z) to the joint types investigated, as will be shown in the present paper.

South African Standard (SABS 0162-1:1993)

According to Clause 13.6 of the SABS 0162-1:1993 the critical elastic moment of an unbraced member is given by:

$$M_{cr} = \frac{\omega_2 \pi}{KL} \sqrt{EI_y GJ + (\pi E / KL)^2 I_y C_w} \qquad (7)$$

where:

KL: effective length of unbraced portion of beam

$\omega_2 = 1,75 + 1,05\kappa + 0,3\kappa^2 \leq 2,5$ (for unbraced lengths subject to end moments)

$\omega_2 = 1,0$ (when bending moment at any point within the unbraced length is greater than the larger end moment or when there is no effective lateral support for the compression flange at one of the ends of the unsupported length)

C_W: Warping torsional constant

κ: Ratio of smaller to larger ultimate moment at opposite ends of the unbraced length (positive for double curvature and negative for single curvature)

The degree of warping restraint at the beam ends, which is highly dependent on the type of beam-to-column connection, is simplified. When beam ends are not restrained against torsion, the values of the effective length in table 2 shall be increased by 20 %. Restraint against torsion can be assumed to be provided by

- web or flange cleats
- load-bearing stiffeners acting in conjunction with bearing of the beam
- lateral-end frames or other external supports to the ends of the compression flanges
- the flanges being built into walls

These assumptions do not specify the degree of torsional restraint in relationship to joint type and could lead to overconservative and thus uneconomic results.

RELATIONSHIP BETWEEN STABILITY BEHAVIOUR AND JOINT DESIGN

Design and construction of frame joints

The continuity at the joints results involves all frame members in the stability behaviour of the frame, and it is this interaction between adjacent members which depends on joint details that complicates the accurate prediction of the lateral-torsional buckling load.

Connections used in frames should have sufficient strength and adequate

Table 1 Critical loads q_{cr} [kN/m] for a beam of single-storey frame

EMM	$\beta_0 = 0,5$	$\beta_0 = 0,6$	$\beta_0 = 0,7$	$\beta_0 = 0,8$	$\beta_0 = 0,9$	$\beta_0 = 1,0$
$\beta_z = 1,0$	4,44	4,20	4,05	3,95	3,89	3,84
$\beta_z = 0,8$	5,15	4,86	4,68	4,56	4,48	4,42
$\beta_z = 0,6$	6,10	5,74	5,51	5,35	5,25	5,18
$\beta_z = 0,5$	6,68	6,26	6,02	5,85	5,73	5,63

(B = 20 m, H = 8 m) column and beam sections IPE 500 using equation 4 for different conditions of support

Table 2 Effective length factor for simply supported beams (SABS 0162-1:1993)

1	2	3
Restraint against lateral bending at supports	Effective length factor K	
	Loading condition	
	Normal	Destabilising[1]
Unrestrained (ie free to rotate in plan)	1,0	1,2
Partially restrained (ie positive connection by flange cleats or end plates)	0,85	1,0
Practically fixed (ie not free to rotate in plan)	0,7	0,85
1 The destabilising loading condition applies when the load is applied to the compression flange of the beam and both the load and the flange are free to move laterally		

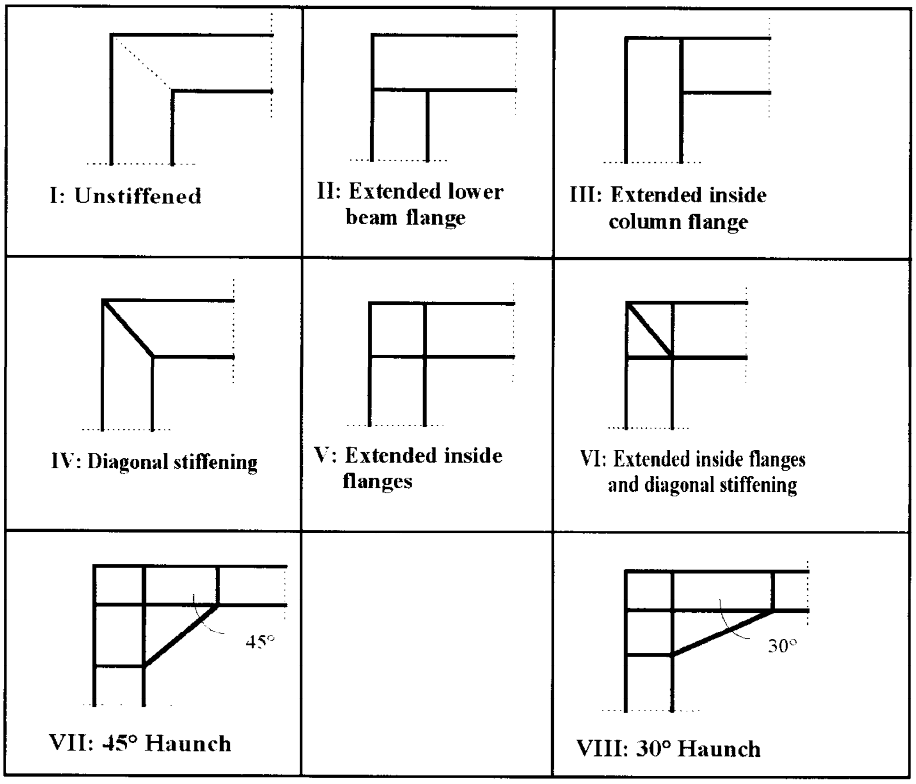


Figure 2 Types of frame joints

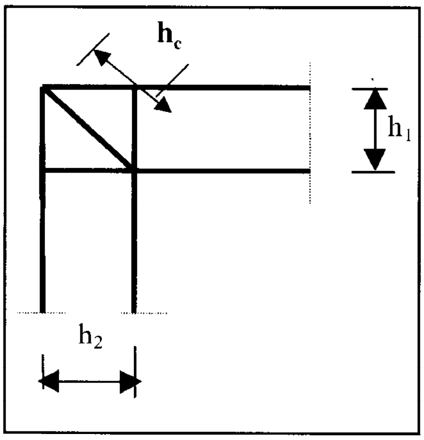


Figure 3 Joint with diagonal stiffener

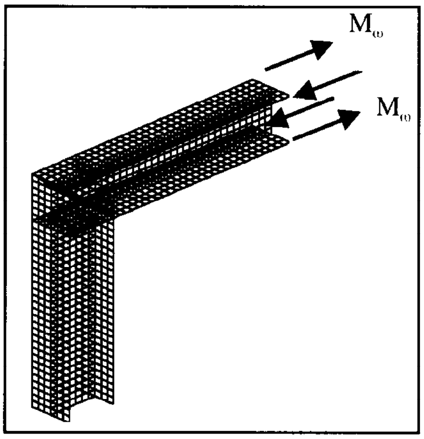


Figure 4 Applied warping moments

overall stiffness in the elastic range. According to Chen and Sohal (1995), the compatibility of warping deformations in one member, with the cross-section distortion in the perpendicular member due to bimoments should be accounted for.

Owing to the continuity of the jointed flanges and the local character of the cross-sectional distortion, the distortion deformation can be expressed in terms of the warping parameters of the two beams at the joints. Krenk and Damkilde (1991) developed a simple theory for the coupled warping and cross-sectional distortion at joints between thin-walled I-beams. On the basis of this theory, the two elastic energy components associated with warping and distortion at beam ends can be approximated. The continuity and stiffness conditions for the frame joints in figure 2 were formulated using the expressions for the elastic energy for warping E_ϑ and that for distortion E_ψ. The author (1997) makes a more comprehensive presentation of the energy components for the various joints. For a joint with a diagonal stiffener (fig 3) the energy components associated with the warping and distortion are:

$$E_\vartheta = \frac{G}{6}\left(\frac{h_2 b_1}{\sin\alpha}t_1^3 + \frac{h_1 b_2}{\sin\alpha}t_2^3 + h_c b_c t_c^3\right)\vartheta_c^2 \qquad (8)$$

$$E_\psi = \frac{1}{2}\left\{\left(\frac{h_1\cos\alpha + h_2}{\sin\alpha}\right)^2\sqrt{\frac{D_{w1}}{2h_1}GK_{f1}} + \left(\frac{h_2\cos\alpha + h_1}{\sin\alpha}\right)^2\sqrt{\frac{D_{w2}}{2h_2}GK_{f2}}\right\}\vartheta_c^2 + \frac{1}{2}\left[a_1^2\sqrt{\frac{D_{w1}}{2h_1}GK_{f1}} + a_1^2\sqrt{\frac{D_{w2}}{2h_2}GK_{f2}}\right]\vartheta_c^2 \qquad (9)$$

where G is the shear modulus; K_f is Saint Venant's torsion constant for full-beam cross-section; h_1, h_2, h_c are the heights of the two beams and of the stiffener respectively; b_1, b_2, b_c are the widths of flanges and of the diagonal stiffener; t_1, t_2, t_c are the thicknesses of the flanges and of the diagonal stiffener; D_{W1}, D_{W2} are the bending stiffnesses of the webs; α is the angle of beam-column connection (ie 90°); $a_1 = (h_2 - h_1)/\sin\alpha$; and ϑ is a parameter for warping intensity.

The sum of the two energy components is

$$E_\vartheta + E_\psi = \tfrac{1}{2}K.\ \vartheta^2 \qquad (10)$$

where

$$K = \frac{G}{3}\left(\frac{h_2 b_1}{\sin\alpha}t_1^3 + \frac{h_1 b_2}{\sin\alpha}t_2^3 + h_c b_c t_c^3\right) + \left\{\left(\frac{h_1\cos\alpha + h_2}{\sin\alpha}\right)^2\sqrt{\frac{D_{w1}}{2h_1}GK_{f1}} + \left(\frac{h_2\cos\alpha + h_1}{\sin\alpha}\right)^2\sqrt{\frac{D_{w2}}{2h_2}GK_{f2}}\right\} + \left[a_1^2\sqrt{\frac{D_{w1}}{2h_1}GK_{f1}} + a_1^2\sqrt{\frac{D_{w2}}{2h_2}GK_{f2}}\right] \qquad (11)$$

The K-value, which is a function of joint design and dimensions, can be regarded as a practical approximation of the warping stiffness of the joint. Different joint types (fig 2) were modelled and analysed using FEMAS 90 (1991). FEMAS 90 is a finite element program which applies both triangular and quadrilateral flat-shell elements (MPLT3 and MPLT4), and was developed at the Bergische Universitaet in Wuppertal (Germany). To verify its capability and reliability, a variety of calcula-

Table 3 C_ω and K-values for joint types (I-sections HEA 400)

Joint type	I	II	III	IV	V	VI	VII (20°)	VIII (30°)	IX (45°)	X (60°)
$[C_\omega]$ kNm³	79,4	107,7	102,6	182,8	213,3	367,8	438,2	518,0	656,5	481,7
[K] kNm³	87,2	106,6	106,6	179,6	213,2	393,0	426,6	503,8	637,8	473,6

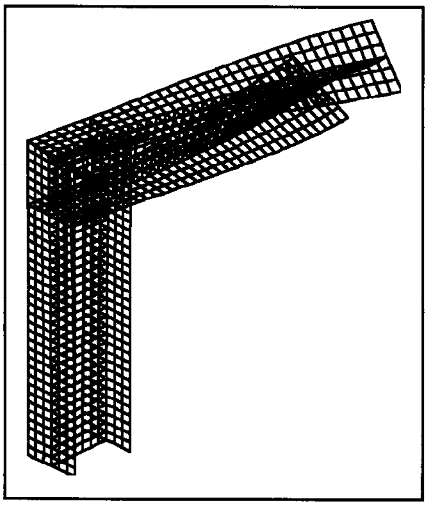


Figure 5 Deformation of joint under warping moments

tions to determine the critical buckling loads of simple frames were conducted with both FEMAS 90 and ANYSYS 53 (Finite Element Program System). The ANYSYS 53 and FEMAS 90 solutions varied from each other by less than 5 %. The optimal degree of discretisation was determined through a series of iterations for the convergence of the finite-element solution.

The effect of joint type on warping of beams was analysed through an application of axial warping forces in the upper and lower flanges (fig 4). These forces are a simulation of bimoments (M_ω), which cause the warping of the flanges of a beam in torsion. This results in the twisting and warping of the beam cross-section (fig 5). Analogous to the rotational stiffness of beam ends, a causal relationship exists between the warping stiffness (C_ω) and the applied warping bimoment (M_ω), that is, $C_\omega = M_\omega / \vartheta'$, where ϑ' is the resultant cross-sectional warping deformation per unit length of beam. This rate of warping deformation (ϑ') in the beam is a direct function of the stiffness of the beam end, that is, joint type. The values of ϑ' can easily be interpreted from the output data of the FEMAS 90 program. Since M_ω is the known (applied) bimoment, the values of C_ω for the various joint types can be calculated. Using equations (8) to (11), the K-values, which are regarded as approximations of the joint warping stiffness, were calculated for the same joint types under investigation. From table 3 it can be observed that the warping stiffness value C_ω derived for each type of joint using finite-element modelling are comparable with the K-values obtained using the energy components for warping and distortion.

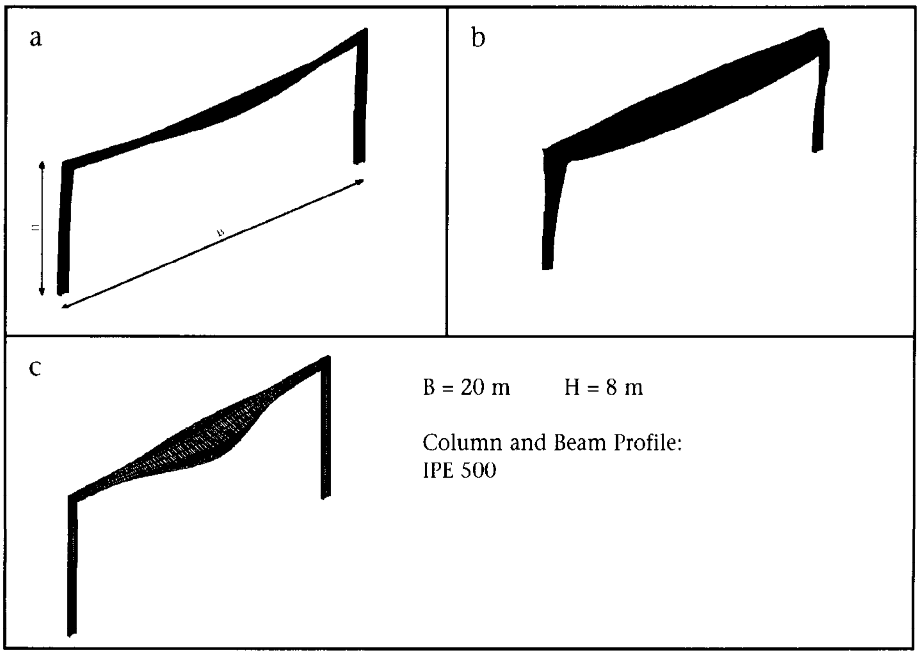


Figure 6 Lateral torsional buckling of frame: (a) lateral restraints at beam ends (b) joint type I and lateral restraints 2,5 m apart (c) joint type VI and lateral restraints 2,5 m apart

Finite-element frame model and lateral-torsional buckling

The effects of joint design on lateral-torsional buckling behaviour of frames were evaluated through the comparison of critical loads. Whole frames with various spans (B) and heights (H), and of the configuration shown in figure 6, were modelled using FEMAS 90, and the critical loads for lateral-torsional buckling

Table 4 Critical loads q_{cr} [kN/m] for lateral-torsional buckling of beam B = 20 m, H = 8 m, roof slope 0°, column and beam sections IPE 500 (steel frame)

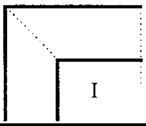
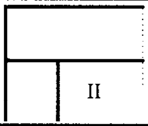
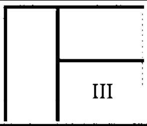
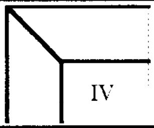
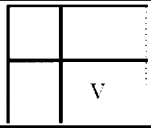
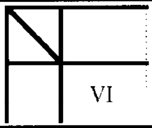
	Bracing system								
Frame joint (type)	P1 Pinned column bases Lateral restraints at beam ends	P2 Pinned column bases a = 5,0 m H/2	P3 Fixed column bases a = 5,0 m H/2	P4 Pinned column bases a = 2,5 m H/2	P5 Fixed column bases A = 2,5 m H/2	P6 Pinned column bases a = B/10 = 2,0 m H/2	P7 Pinned column bases a = B/10 = 2,0 m H/3	P8 Pinned column bases a = 1,0 m H/2	P9 Pinned column bases a = 1,0 m H/3
I	3,20	3,91	3,95	3,97	3,98	3,98	4,00	8,30	8,36
II	3,88	12,85	13,23	13,99	14,39	14,41	14,52	14,54	14,61
III	4,07	16,67	17,25	19,68	20,03	20,09	20,21	20,27	20,38
IV	4,63	17,20	17,93	21,73	22,31	23,01	23,11	23,61	23,63
V	4,61	18,53	19,13	20,78	21,39	21,57	21,59	21,96	21,99
VI	5,28	22,33	23,01	35,57	36,91	37,07	37,31	39,59	39,72
VII 45° Haunch	6,63	25,03	26,22	40,53	42,36	43,67	44,57	44,71	45,99
VIII 30° Haunch	7,46	29,70	32,08	42,93	44,12	44,43	45,30	45,51	46,80

a: Distance between lateral restraints H/2, H/3: Distance between girts on outer column flanges

Table 5 Warping and bending restraint coefficients assigned to joint types from Table 3 (beam B = 20 m, H = 8 m, roof slope 0°, column and beam sections IPE 500 (steel frame)

Joint type	I	II	III	IV	V	VI	Haunched joints VII / VIII
β_0	*	1,0	0,9	0,7	0,7	0,6	0,5
β_z	*	1,0	1,0	0,8	0,8	0,7	0,5

Table 6 β_0- and β_z-values for joint types in different portal frames

Portal Frame	β Values	Joint type					
		 I	 II	 III	 IV	 V	 VI
A B = 10 m Beam: IPE500	β_0	1,0	0,9	0,9	0,6	0,6	0,5
H = 4 m Column: IPE500	β_z	0,9	0,9	0,9	0,7	0,6	0,6
B B = 10 m Beam: IPE500	β_0	1,0	0,9	0,9	0,6	0,6	0,5
H = 6 m Column: IPE500	β_z	0,9	0,8	0,8	0,7	0,7	0,6
C B = 12,5 m Beam: IPE500	β_0	*	0,8	0,8	0,6	0,6	0,6
H = 4 m Column: IPE500	β_z	*	0,8	0,8	0,7	0,7	0,5
D B = 15 m Beam: IPE500	β_0	0,9	0,9	0,9	0,7	0,6	0,5
H = 5 m Column: IPE500	β_z	0,7	0,8	0,8	0,7	0,7	0,6
E B = 15 m Beam: IPE500	β_0	0,9	0,9	0,8	0,6	0,6	0,5
H = 5 m Column: HEA500	β_z	0,7	0,8	0,7	0,7	0,6	0,5

B = Span of frame H = Height of frame
* Stability failure due to local buckling

Table 7 Increment factor γ of the critical loads q_{cr} due to the reduction of lateral restraints spacing from B to a for frame A

	Increment factor γ of q_{cr} for different Spacings a between the lateral restraints							
Spacing	a = 5,0 m		a = 2,5 m		a = B/10		a = 1,0 m	
Joint type	I	VIII	I	VIII	I	VIII	I	VIII
Factor γ	1,2	4,3	1,2	7,0	1,2	7,1	1,2	7,5

determined for different joint types, frame dimensions, profile sections and bracing systems. The fixity or non-fixity of the column bases, the position of girts along the outer column flanges (H/2 or H/3), and lateral restraints (eg purlins) on beam upper flange spaced at the distance a apart, were considered. For simplicity, one loading case in the form of a vertical uniformly distributed loading applied on the beam, was used. The critical loads for one of the investigated frames with different joint types are shown in table 4. These results can consequently be used to provide a practical and vital approximation of the influence of joint type on the lateral-torsional buckling load of a frame. Since the critical loads in each column of table 4 are those of the same frame with different joint types, the variations in the lateral-torsional buckling loads are solely due to the joint type used.

Through the comparison of the critical loads obtained from the finite-element method (table 4, column P1) with those calculated on the basis of the equivalent-member method for all possible combinations of β_0 and β_z values (table 1), a practical proposal for a specific β_0–β_z combination for each joint type in figure 2 is made. Therefore, warping (β_0) and bending (β_z) restraint coefficients can be assigned to each joint type (table 5). The DIN standards [3] and [4] do not specify the β_0 and β_z coefficients for the joint type and leave this determination of the joint effect on the stability behaviour of framed structures to the engineer. The assigning of specific β_0 and β_z coefficients to various joint types, as proposed in this paper, is a useful and practical solution to the dilemma faced by many engineers in dealing with the stability problems of such structures.

It should, however, be stated that a comparison of critical loads in tables 1 and 4, as proposed in this paper, could possibly lead to the assignment of more than one combination of β_0 and β_z coefficients to a specific joint type. This absence of a unique solution is, however, expected, since the joint type has an effect on both the warping and rotation at the beam end, and various combinations of these effects could lead to similar lateral-torsional buckling loads of the frame. But the difference between the possible values of β_0 and β_z coefficients for a specific joint type is relatively small and the analysis results still accurate. The critical load from the finite-element model for this particular frame with joint type I (unstiffened joint) lies below the smallest value obtained by the equivalent-member method and failure is due to local effects in joint. Assigning β_0 and β_z values to this joint is therefore not critical and might even be misleading.

There are numerous parameters of the frame that have an influence on structural behaviour, and the joint type is only one of them. Some of the vital characteristics are frame geometry and the section profiles selected for the beam and columns. In order to determine the effect of some of these parameters, lateral-torsional buckling loads for various portal frames (A-E) with different dimensions (B, H) and section profiles (eg IPE 500, HEA 500) were calculated and β_0 and β_z coefficients assigned to the joint types (table 6). The results in table 6 are just a selection from the various calculations conducted by the author. A more comprehensive list of the different portal frames investigated can be found in Masarira (1997).

It is evident that both the frame geometry and the member sections have an effect on the stability behaviour of the structure. Frames with large beam span or column height have a relatively lower critical load (ie less stable) than compact frames and larger sections, which lead to more stable structures. A trend can also be observed in the values of the β coefficients, indicating the differences in stiffness between the relatively weak joints (type I, II) and the relatively stiff ones (type VI). Taking all three parameters into consideration, that is joint type, frame geometry and the member section, the joint types can be placed into three major stiffness categories.

The joint types I to III can be considered weak, joints IV and V are of medium stiffness, while joint VI and the haunched joints (VII, VIII) are stiff joints. The β coefficients of the weak joints range from 0,8 to 1,0, while those of the medium joints are about 0,6 and 0,7. The stiff joints can be assumed to have β coefficients of 0,5 or 0,6.

As expected, lateral restraints on the upper flange of the beam result in more rigid frames with higher stability loads which are inversely proportional to the distance (a) between the lateral restraints. The design and the stiffness of the joint plays an increasingly important role when a decreases (table 7).

CONCLUSIONS

Given that joint design plays an important role in the lateral-torsional buckling assessment of simple steel frames, a realistic modelling of the support-joint conditions is necessary for both economy and safety. Approximations of the conditions concerning warping at the beam ends of frames, according to the standards of most countries, appear inadequate in the assessment of the effect of most joints on the stability behaviour of frames. This study is an endeavour to fill this gap, by proposing simple coefficients that can be used in approximating the joint effect on the lateral-torsional buckling load of steel frames. As described above, the warping behaviour at the beam ends depends on the joint type and has an influence on the stability of the frame. By developing a more accurate estimate of the relative joint-warping stiffness, this study contributes towards a more accurate evaluation of the structural stability of frames. The magnitude of the stabilising effect of lateral restraints (eg purlins) and the role of the joints on such frames are evident, and to ignore these effects could be uneconomical. Although the frame geometry has an effect on the presented results, it is evident that the warping stiffness values of the joints are a function of the joint design and joint parameters, as is evident from the results in table 6.

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