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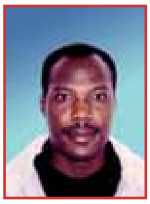
Pump and reservoir system operational optimisation using a non-linear tool

A A Ilemobade, N J Manson and D Stephenson

This paper presents a methodology for optimising the pumping policy of a pump and reservoir system over the short as well as the long term using the downhill simplex method (DSM). The pumping policy fits pumping rates into a minimum cost cycle in order that reservoir volumes are kept within prescribed limits over a given period. This requires knowledge of reservoir storage capacities, consumer demands, pumping capacities and electricity tariffs, which are often a function of time of use. Cost penalties are applied when reservoir limits are violated. The DSM-generated pumping policy is validated using a simple dynamic programming (DP) module. The DP module, using mass-balance, is capable of generating a near global least cost solution based on historical or synthetic demand data, but it does not generate a pumping policy that can be used in operating the system over short- and long-term periods. The DSM, on the other hand, is capable of generating an optimum pumping policy for such durations, but requires verification to ascertain if the cost generated by the optimal pumping policy is the least possible cost – hence the use of the DP module. In tandem, these two methods provide an effective tool for pump and reservoir system operation.



ADESOLA A ILEMOBADE BEng, MEng, PhD currently lectures in the School of Civil and Environmental Engineering at the University of the Witwatersrand. He researches and teaches water and urban engineering at both undergraduate and postgraduate levels and his current research interests are developing decision support systems for water supply to especially rural and peri-urban communities, and engineering education.

NEIL J MANSON BSc (Mech Eng), MSc (Civil Eng) lectures in optimisation, simulation and programming at the School of Information Technology, Monash University South Africa. He was formerly a member of the Water Systems Research Group at the University of the Witwatersrand. His current research interests are in distributed simulation and optimisation models, as well as free and open source software.

DAVID STEPHENSON BSc(Eng), MSc(Eng), PhD, DSc(Eng) PrEng, FSAICE is Professor of Water Engineering at the University of Botswana and former Rand Water Professor of Water Engineering and Director, Water Systems Research Group, University of the Witwatersrand. He has extensive experience with Rand Water and as partner in an international consulting firm. He has published over 150 papers and eight books. He is active in the fields of water supply, water resources, hydraulics, hydrological modelling and water hammer.

Keywords: pumping policy, cost, downhill simplex

INTRODUCTION

Pump and reservoir system operators are constantly faced with the need to make regular decisions as to which pumps should operate at any given time, determine the rate of pumping in order to meet varying demands, ensure that reservoirs do not drain, and minimise costs. Decision-making based on heuristics alone is the trend in the operation of many water supply systems that serve municipalities. Without doubt, this can be improved by employing in tandem, more deterministic tools. In respect of ensuring adequate reservoir storage, water supply operators in southern Africa are legally and socially bound to supply their consumers with adequate potable water when required. Reneging on existing agreements by allowing reservoirs to drain may incur unpleasant legal and related costs to the water supplier. Pumping energy costs, on the other hand, form an important part of the operational cost of water distribution systems worldwide (Van Zyl *et al* 2004). They are usually higher than water purification costs, particularly in Gauteng Province, South Africa, where the consumer is at a considerably higher elevation than one of its major water sources, namely the Vaal River.

Multidimensional optimisation of the variables mentioned above has become necessary to maximise system efficiency and minimise capital and recurrent costs (Ormsbee & Lansey 1994). A small overall increase in operational efficiency could result in significant cost savings to the water industry. Theoretical studies and practical implementation of optimal pump scheduling in various types of supply systems suggest that 10 % of the annual expenditure on energy and related costs may be saved if proper optimisation methods are used (Mackle *et al* 1995). Important features of cost are the electricity tariff structure, the relative efficiencies of the available pump sets, the head through which they pump, and marginal treatment costs. Important constraints include physical system limitations (reservoir capacities, abstraction limits, pumping capacity, treatment works throughput), physical laws (conservation of mass and energy laws) and externally defined requirements (consumer demands, politics) (Ormsbee & Lansey 1994). Additional benefits of operational optimisation include improved water preservation and quality, ensuring compliance with water industry regulations, improved system management, and benefits for future expansion such as automation (Jarrige *et al* 1991).

The problem of finding the optimal operating strategy is far from simple (Van Zyl *et al* 2004):

- Both consumer demand and the electricity tariff can vary greatly through a typical operating cycle – electricity tariffs are varied in an attempt by power suppliers to shed load or distribute the load more evenly in order to operate at as high a load factor – that is, the ratio of the actual energy consumption (kW.hr) to the maximum power recorded (demand) over a period of time – as possible and minimise electricity costs. As a result, the user of energy is encouraged to use off-peak energy with a preferential tariff system.

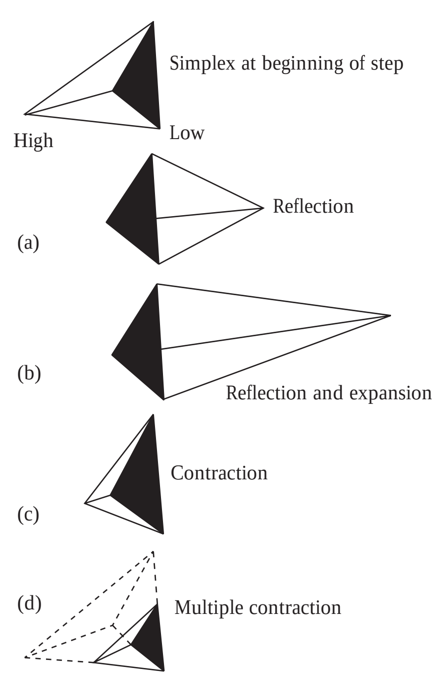


Figure 1 The movements of the downhill simplex method

- Minimum and maximum levels of water have to be maintained in the reservoirs to ensure reliability of the supply – if pumping rates are too low, the reservoirs may drain during peak demand, and if pumping rates are too high, the reservoirs may overflow, resulting in wasted electricity costs.
- The number of pump switches in an operating cycle has to be limited to avoid excess pump operating and maintenance costs.
- The number of possible operating strategies becomes vast for systems with more than a few pumps and reservoirs. Added to the above is the fact that the hydraulic behaviour of water distribution systems is highly non-linear, making computer modelling a complex and time-consuming process.

A logical solution to these challenges is to develop a set of rules that would guide system operation, maximise efficiency and minimise costs for a particular system configuration and demand characteristics. This is a complex problem to solve – as indicated above – and has been attempted by several workers using diverse mathematical tools: Lansey and Awumah (1994), Sakarya and Mays (2000) – nonlinear programming; Zessler and Shamir (1989), McCormick and Powell (2003) – dynamic programming; Angel *et al* (1999) – fuzzy logic; and Mackle *et al* (1995), Schwab *et al* (1996), Van Zyl *et al* (2004) – genetic algorithms.

Drawbacks of many of these models include:

- their inability to determine pumping schedules for a system over the short-term (eg one day) as well as long-term (eg one month) durations if system configuration and demand characteristics remain unchanged
- their insistence that reservoirs must finish their daily cycle at the same level from which they started in order that adequate volumes are maintained in the reservoir for balancing storage purposes
- their simplification of the optimisation problem through assumptions, discretisation or heuristic rules. Such simplification makes it easier for specific optimisation methods to determine the optimal solution, but introduces bias into the solution by excluding a larger number of potentially good solutions (Van Zyl *et al* 2004)
- their ability to work well for small systems but encounter difficulties when dealing with larger ones. For example, they may become inadequate when there are more than two reservoirs in the system or even for one-reservoir systems which have several different pump combinations or complicated system constraints

This paper tackles the pump and reservoir operation problem by utilising a multi-dimensional non-linear tool, the downhill simplex method (DSM), to determine an optimum pumping policy for a system based on demands, pumping rates and reservoir volumes (historic or predicted) over short- as well as long-term periods. As long as reservoir volumes remain within specified limits, it is assumed that supply and minimum required pressures are guaranteed within the water reticulation network. Most municipal systems are operated in a manner that keeps the reservoirs as full (or as empty) as possible all the time at the minimum risk to the consumer. This culture is incorporated into the DSM using a storage cost function. In addition, electricity tariffs, pump switching costs and storage penalty costs are considered when determining a system's optimal pumping policy. A pumping policy refers to the set of rules scheduling pumping operations at different specified reservoir levels (volumes) that will result in the minimum operating costs for a given period and operating conditions (Ormsbee & Lansey 1994; Mackle *et al* 1995). The pump and reservoir system looked at is essentially a pumping main from a source of water with an unlimited capacity, multiple pumps, and supply into storage reservoirs at the head of the demand system. In this optimisation problem, the objectives are the minimisation of pumping costs, penalties for violating reservoir limits, storage costs and pump switches.

The DSM is a multi-variable, constrained, non-linear optimisation tool, employed to determine a system's optimum pumping policy. By applying the policy, operating costs vis-à-vis pumping, pump switching, storage and penalty costs are determined over the given period. The optimum pumping policy is validated using dynamic programming (DP). This is a simpler non-linear method, confined to using mass-balance for optimising only the overall pump flow rate on sequential time periods so as to obtain a near global least cost solution – it does not generate a pumping policy. Since DP is able to determine a near global least cost solution over the short- and long-term durations required, it is employed to confirm whether the DSM has located an optimal pumping policy whose operating costs are close to its least cost solution.

The DSM determines a pumping policy using analysed historical or synthetic demands. After analysis, demands are predicted. As long as predicted demands are statistically consistent with its original demand set, a pumping policy generated using the DSM remains optimal. If there are significant variations between predicted and actual demands or changes to system components, a new policy will need to be recalculated. As is typical of a non-linear method, the DSM requires a number of runs from different starting points before an optimum pumping policy can be achieved, as there may otherwise be local optima that confine the optimisation.

The tool presented herein is useful for manually operated and semi-automated pumping systems, especially in developing communities, as it generates formal guidelines for pump scheduling that facilitate optimal pumping at the least operating costs. It is not the intention of the authors to present a 'gold standard' for system operation, but a practical and usable solution for small pumping system operations.

APPLICATION OF THE DOWNHILL SIMPLEX METHOD IN DETERMINING A PUMPING POLICY

The DSM (Nelder & Mead 1965; Press *et al* 1992; Koshel 2002) finds the minimum value of a function that has more than one independent variable. It only requires function evaluations and not derivatives. It is very efficient for constrained coefficient optimisation and especially for problems that have a small computational burden (< 20 dimensions). While it is a robust method of optimisation, it is relatively slow to converge to local minima for larger computations. However, its stability, lack of use of derivates and efficiency with small computational problems (a pump and reservoir system is unlikely to contain 20 pump settings) make it very appropriate for the pumping policy problem.

A simplex is an N-dimensional geometrical figure, having $N + 1$ vertices connected with straight lines. In figure 1, the two-dimension simplex is a triangle (N = 3). In three dimensions it is a tetrahedron. The DSM starts from $N + 1$ points, defining an initial simplex. For one initial point P_0 on the simplex, the other N points can be expressed by:

Table 1 Diverse spread of starting points for example system in figure 2

Starting points	Reservoir change levels (Mℓ)		
	X	Y	Z
Random	Random	Random	Random
Minimum	Min	Min	Min
Even	$\frac{1(Max - Min)}{N+1}$	$\frac{2(Max - Min)}{N+1}$	$\frac{3(Max - Min)}{N+1}$
Maximum	Max	Max	Max
Centre	$\frac{N(Max - Min) + 2Max}{N(N+1)}$	$\frac{N(Max - Min) + 3Max - Min}{N(N+1)}$	$\frac{N(Max - Min) + 4Max - 2Min}{N(N+1)}$

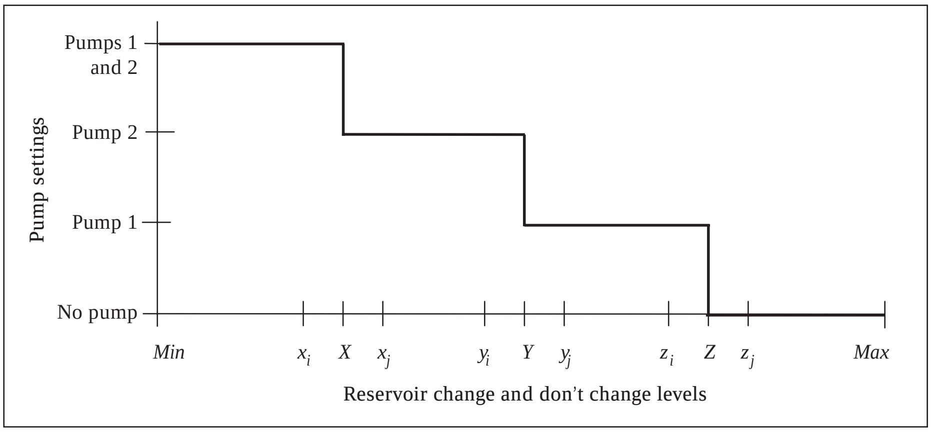


Figure 2 Pumping policy for an example pump and reservoir system

$$P_i = P_0 + a_i e_i \quad (1)$$

where e_i represents N unit vectors, and a_i represents constants that characterise the length scale for each vector direction.

During optimisation, the general idea is to keep the minimum solution within the simplex while at the same time decreasing the volume of the simplex. The DSM essentially has four possible steps during each iteration: *reflection*, *expansion*, *contraction in one dimension* and *contraction around the low vertex*. The basis for each step is provided below:

1 The vertices of the initial simplex that have the highest (x_h), second highest (x_s) and lowest (x_l) function values are determined. The centroid x_0 of all x except x_h is also determined.

2 x_h is *reflected* through x_0 to x_r (equation 2). If $f(x_s) > f(x_r) > f(x_l)$, x_h is replaced with x_r and return to step 1 else step 3 or 4.

$$x_r = (1 + \alpha)x_0 - \alpha x_h \quad (2)$$

Where α is the *reflection factor* (Nelder & Mead 1965: $\alpha = 1$)

3 If $f(x_r) < f(x_l)$, the simplex is *expanded* (equation 3) along the x_0 direction. If *expansion* is successful (ie if $f(x_e) < f(x_l)$), x_h is replaced by x_e and return to step 1.

$$x_e = \gamma x_r + (1 - \gamma)x_0 \quad (3)$$

Where γ is the *expansion factor* (Nelder & Mead 1965: $\gamma = 2$)

4 If $f(x_r) > f(x_s)$, *contraction in one dimension* (equation 4) is made along the x_0 direction.

$$x_c = \beta_i x_j + (1 - \beta)x_0 \quad (4)$$

Where β_i is the *contraction in one dimension factor* (Nelder & Mead 1965: $\beta_i = 0{,}5$). x_j is the selection of x_h or x_r, whichever has the lowest function value.

If $f(x_j) > f(x_c)$, x_j is replaced by x_c and return to step 1. If not (ie if $f(x_j) < f(x_c)$), the simplex is *contracted around the low vertex* (equation 5) about the point x_l and return to step 1.

$$x_i = (1 - \beta_2)\, x_l + \beta_2 x_i \quad (5)$$

Where β_2 is the *full contraction factor* (Nelder & Mead 1965: $\beta_2 = 0{,}5$) and x_i represents all the points except x_l.

Typically, when a point replaces x_l, the current iteration is completed. Next the termination condition is checked. If the tolerance is not met, then the next iteration is started. If the tolerance is met or the number of function evaluations exceed some threshold value (unsuccessful termination), the optimisation is done. It is frequently a good idea to restart the algorithm at the point where it claims to have found a minimum solution. Maintaining a family of diverse (eg table 1) yet near optimal solutions as a starting point for the evolution may allow rapid identification of the optimum pumping policy (Mackle *et al* 1995).

A pumping policy decision is determined based on the reservoir volume at the start of a time step, the demands expected for that time step, and the pump settings during the previous time step. *Pump settings* refer to various combinations of pumps and thus, pump flow rates. A *reservoir change level* is assigned to each *pump setting* – it specifies the maximum value of the predicted reservoir volume for which a pump setting may be used. A predicted reservoir volume is defined as current reservoir volume minus predicted demands. *Reservoir don't change levels* are also defined for each *pump setting* in the DSM – these *change levels* prevent pump switching or changing from one pump setting to another in instances where the predicted reservoir volume is only just outside a *reservoir change level*.

The DSM is initially given a starting guess, that is, a simplex with *N + 1 reservoir change levels* all attached to different *pump settings*. It then finds the system's optimum pumping policy (ie each *reservoir change level* and *reservoir don't change level* that minimises operating costs over a given period).

Figure 2 illustrates the pumping policy for an example pump and reservoir system having two constant flow pumps with three pump settings *3, 2 and 1* (excluding pump setting *0*) and therefore three *reservoir change levels X, Y and Z*. Variable speed pumps may be modelled as a number of discrete settings. Each *reservoir change level* is the maximum *predicted reservoir volume* at which a pump setting may be used, for example pump setting *1* is recommended for operation if the *predicted reservoir volume* is between *reservoir change levels Y* and *Z*. x_i, x_j, y_i, y_j, z_i and z_j represent reservoir do not change levels. Incorporating these into the example above, pump setting 1 is recommended for operation between y_i and z_j. Any pair of X:Y:Z values will therefore represent a starting point on a three-dimensional plane. The optimum pumping policy will then find the values of x_i, X, x_j, y_i, Y, y_j, z_i, Z and z_j that will result in the least system operating costs over a specified period of operation.

By employing the Darcy-Weisbach formula, total system head H_{system}, which is a function of flow rate Q, may be mathematically represented as:

$$H_{system} = H_{elevation} + H_{reservoir} + \Delta H_f + \Delta H_m \quad (6)$$

$H_{elevation}$ is the height of reservoir base above pump discharge, $H_{reservoir}$ is height of water in reservoir, ΔH_f is headloss due to pipe friction, and ΔH_m is secondary headloss.

$$\Delta H_f + \Delta H_m = \frac{8\lambda L Q^2}{\pi^2 g D^5} + k_L \frac{8Q^2}{\pi^2 g D^4} \quad (7)$$

$$\Delta H_f + \Delta H_m \cong KQ^2 \quad (8)$$

λ is the Darcy friction coefficient, L is pipe length, D is pipe diameter, and g is acceleration due to gravity. K_L is minor loss coefficient, K, which incorporates the parameters above, is approximately constant for an installed pumping mains flowing fully.

For each pump setting, a curve relating pump head H_{pump} to flow rate Q may be expressed as:

$$H_{pump} = f(Q) \quad (9)$$

$f(Q)$ is usually approximated as a cubic or square polynomial.

At a system's operating point:

$$H_{pump} = H_{system} \quad (10)$$

By substituting equations 6 and 9 into 10, and rearranging,

$$Q = f(V) \quad (11)$$

For each pump setting, therefore, there exists an equation that relates pump flow rate Q to reservoir volume V.

Operating cost components

Pump maintenance is an important operating cost component in providing potable water. Although the actual cost of wear on the pumps due to pump switching cannot be easily quantified, it can be reasonably assumed that it increases as the number of pump switches increases (Ormsbee & Lansey 1994). A pump switch is 'turning on a pump that was not operating in the previous period' (Lansey & Awumah 1994). Associated with pump switching is the peak demand charge (rand/KW). The peak demand charge is the charge for the highest power delivered during peak periods. The switching-on of pumps during peak demand periods increases this charge. It makes economic sense therefore to also regulate the switching-on and off of pumps during peak periods. In generating an optimal pumping policy, the DSM uses the *reservoir don't change level* variable to minimise pump switching and consequently, pump maintenance and peak demand charges.

For the active energy charge (cents/KW.hr), Eskom bills clients peak, standard and off-peak rates on several tariff systems (Eskom 1995). It is to the advantage of the water supply authority to choose a convenient tariff system that encourages increased pumping during off-peak periods and decreased pumping during peak periods. The optimisation of this charge versus pumping is not considered in the methodology presented here. Pumping costs, $C_{pumping}(P_{s,t})$, are therefore simply calculated as the energy consumed multiplied by the applicable tariff:

$$C_{pumping}\,(P_{s,t}) \;=\; \frac{\rho\, g\, QH_{pump}}{1000\eta}\; xE_{\text{cost}} \quad (12)$$

ρ represents density of water (kg/m^3), η is pump efficiency (%), and E_{cost} is the energy tariff (cents/KW.hr).

A typical municipal reservoir comprises balancing, operational freeboard, bottom and emergency storage volumes. The balancing storage services the day-to-day function of balancing supply versus demand. To prevent the balancing storage volume from encroaching on the other storage components, reservoir limits (*Maximum*, *Minimum*, *High* and *Low*) and cost penalties are specified in the methodology. *Maximum* and *Minimum* limits are structural limits of the reservoirs that cannot be arbitrarily changed. When the reservoir overflows (above *Maximum*), indirect costs may be incurred in water wastage or damages to the surrounding area. When the reservoir empties (below *Minimum*), costs may be incurred from supplementing water from alternative sources or settling contractual obligations with consumers who have been ill-affected by the lack of water. *High* and *Low* limits represent volume boundaries with adjacent storage components – cost penalties may be allocated to violation of these boundaries and these boundaries may be changed.

Another major operating cost considered is that of storing water in the reservoir: this is defined using a storage cost function and is based on the volume in the reservoir at each stage. This cost is primarily related to unused capacity in the reservoir. A negative cost function will encourage the DSM to keep the reservoirs as full as possible and vice versa.

Pump switching, storage penalty and storage costs are at present hard to quantify as each local situation will possess unique values. For this reason, these penalties are hardly accounted as system operating costs (Ormsbee & Lansey 1994; Lansey & Awumah 1994). The DSM pumping policy will attempt to optimise the trade-offs between these costs at each time step to determine the least cost between, for instance, keeping a pump running or switching it off, violating or not violating certain reservoir limits, and keeping a reservoir as full or as empty as possible.

VALIDATING THE DOWNHILL SIMPLEX PUMPING POLICY USING DYNAMIC PROGRAMMING

A simple dynamic programming module is employed to validate the optimum pumping policy determined in the DSM. The DP, using mass-balance, has the capability to determine a near global least cost solution, but is unable to generate a pumping policy that would facilitate short- as well as long-term system operation. The DSM can generate a pumping policy for this purpose, but requires verification that the policy generated will result in the least operating costs, similar to those of the DP. If the operating costs generated in both procedures are similar for the same data set, the pumping policy is considered optimum. The DP analyses the total length of demand data that is available and, using mass-balance, finds the best pump flow rate and therefore least operating cost for each time stage. The overall objective is to minimise operating costs at each stage and this may be expressed as:

$$\sum_{t=1}^{n} C(P_{s,t}, V_t) = \sum_{t=1}^{n} [C_{penalty}(V_t) + C_{storage}(V_t) + C_{pumping}(P_{s,t}) + C_{pump\ switching}(P_{s,t}, P_{s,t+1})] \quad (13)$$

The stage-to-stage transformation equation relating variables in stage t to those in stage $t\text{-}1$ is:

$$V_t = V_{t-1} + P_r(P_{s,t}, V_{t-1}) - D_{f,t} \quad (14)$$

n is the total number of time stages t, $C(P_{s,t}, V_t)$ represents operating cost, $C_{penalty}(V_t)$ represents reservoir penalty costs, $C_{storage}(V_t)$ represents reservoir storage costs, $C_{pumping}(P_{s,t})$ are pumping costs, $C_{pump\ switching}(P_{s,t}, P_{s,t+1})$ are pump switching costs, V_{t-1} and V_t are reservoir volumes at the start and end of stage t respectively, $P_{s,t}$ represents pump setting during stage t, $D_{f,t}$ is predicted demand for stage t, and $P_r(\,)$ calculates pump flow rate as a function of reservoir volume (equation 11).

PREDICTING CONSUMER DEMANDS

For a pumping policy to be optimal over a given period in the present and future, historical and predicted demands must be considered. For prediction purposes, historical demands must be modelled, analysed and disaggregated into the various components (Wheldon & Thirkettle 1985):

$$D_{f,t} \;=\; D_{s,t} + D_{p,t} + \sum_{i-1}^{q} \phi_{i,q}\; D^{'}_{r,t-i} \;+\; \mu_{D_n} \quad (15)$$

The secular trend component $D_{s,t}$ is usually due to effects such as population growth in the demand area. It is estimated using a least squares regression which fits a linear and logarithmic curve to the demand data, and chooses the best fit. A harmonic analysis is employed to calculate periodic trends, $D_{p,t}$. This trend models the annual, weekly and daily cycles in the demand data. The residual component that remains after the secular and periodic trends have been removed is random in nature, but may not be serially independent. A linear autoregressive model is then used to represent the serial correlation between a particular demand value and those of previous time steps. $D^{'}_{r,t-i}$ represents i previously observed demand values. If $t - i < 1$, then these values are set to zero. $\phi_{i,q}$ are autoregressive weights and q is the order of the linear autoregressive model: this order is based on the significance chosen by the user. μ_{Dn} is the mean of the independent random component and is determined by finding the values that result in the best fit between measured and predicted data. This fit is measured using a Chi-squared (χ^2) test.

THE LIBANON SYSTEM CASE STUDY

The Libanon pump and reservoir system is located on the West Rand in Gauteng, within the Rand Water supply system (Rand Water is Gauteng's bulk water supply authority). In order to apply the DSM

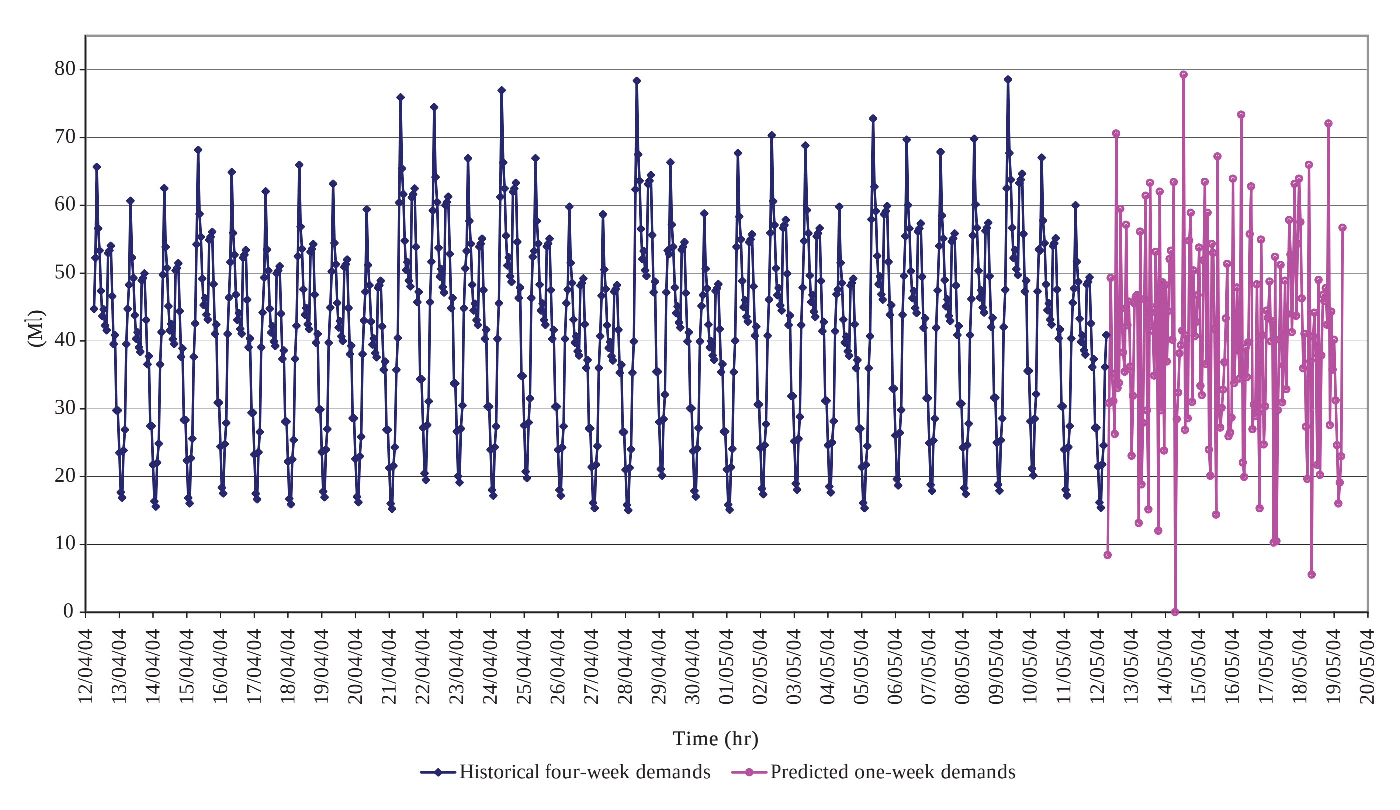


Figure 3 Historical and predicted hourly demands

Table 2 Analysis of hourly and daily demand sets

Classification	Description	Hourly demands		Daily demands	
		Historical (four weeks)	Predicted (one week)	Historical (six months)	Predicted (six months)
Data range	Start date and time End date and time Number of points Minimum demand Maximum demand	12/04/2004 06:00 12/05/2004 06:00 721 15,06 Mℓ/hr 78,58 Mℓ/hr	12/05/2004 07:00 19/05/2004 06:00 168 5,53 Mℓ/hr 79,28 Mℓ/hr	15/11/95 06:00 12/05/96 06:00 180 19,16 Mℓ/day 69,68 Mℓ/day	15/11/95 06:00 12/05/96 06:00 180 37,04 Mℓ/day 68,86 Mℓ/day
Secular Trend Components	Type Slope R^2	Linear 0,0516 0,0011	Log 0,9921 0,0014	Linear -0,0338 0,0478	Linear -0,0375 0,0895
Periodic Trend Components	Significance level Annual cycle Weekly cycle Daily cycle	95,00 % Not calculated 0,1287 0,1062	95,00 % Not calculated Not calculated 1,8141	95,00 % Not calculated 0,0186 Not calculated	95,00 % Not calculated 0,0186 Not calculated
Serial correlations	Coefficients at lag: 0 4	1 0,1524	1 0,2116	1 0,3095	1 0,2153
Autoregressive component	Best-fit $\phi_{1,1}$	0,3420	0,1665	0,5853	0,5658
Best-fit for independent random component	Standard deviation χ^2 significance	10,1649 0,000004283 %	17,0061 15,86 %	4,4554 99,33 %	4,3906 97,73 %

Table 3 Pumping characteristics

Pump settings	Operational pumps	Pumping rate Q as function of reservoir volume V (Mℓ/hr)
1	Pump 1	$Q_1 = +1x10^{-5}V^2 - 0{,}0365V + 36{,}4749$
2	Pump 2	$Q_2 = +2x10^{-5}V^2 - 0{,}0432V + 39{,}3986$
3	Pump 3	$Q_3 = +3x10^{-5}V^2 - 0{,}0594V + 62{,}0174$
4	Pumps 1 and 2	$Q_4 = +1x10^{-5}V^2 - 0{,}0319V + 76{,}7301$
5	Pumps 1 and 3	$Q_5 = -1x10^{-5}V^2 - 0{,}0464V + 88{,}0152$
6	Pumps 2 and 3	$Q_6 = -1x10^{-5}V^2 - 0{,}0471V + 88{,}5881$
7	Pumps 1, 2 and 3	$Q_7 = -1x10^{-5}V^2 - 0{,}0544V + 94{,}0897$

to the Libanon system, demand data over short-term as well as long-term periods were required. Since the system was only operated on a daily basis, short-term (hourly) demands were unavailable. Fictional short-term demands over four weeks (12/04/2004 – 12/05/2004) were therefore generated. Over the long term, actual (daily) demand data (15/11/1995 – 12/05/1996) were used. The short-term demand set was analysed and predicted over one week while the long-term demand set was analysed and predicted over six months. Pumping decisions are made daily at 06:00 and are based on operators' experience and the monthly water supply quotas prescribed by Rand Water. These quotas are based on demand predictions and/or historical averages for the month in question. The operators receive minimal assistance during decision-making. There is no mechanism for determining when and by how much pump settings should change.

From the demand analysis performed (table 2) three demand sets (historical four weeks, historical six months and predicted six months) exhibit a linear secular trend. The slope values indicate increasing or decreasing demands over time. The predicted one-week demand set deviates from the three above by exhibiting a logarithmic secular trend, which implies a nonlinear increase in demand over time. The longer the predicted demands are, the more significant the secular trend becomes: this is evidenced by the larger R^2 values for the monthly demand sets. In the periodic trend component about four times more data points are required for annual, weekly or daily cycles to be calculated. As a result, annual cycles were not calculated for all sets. The monthly demand sets have serial correlations of greater than 21,00 % at lags of up to four days. Although serial correlations appear consistent at lag 1 for the monthly demand sets, they decrease rapidly afterwards because of the first-order autoregressive model used in the calculations.

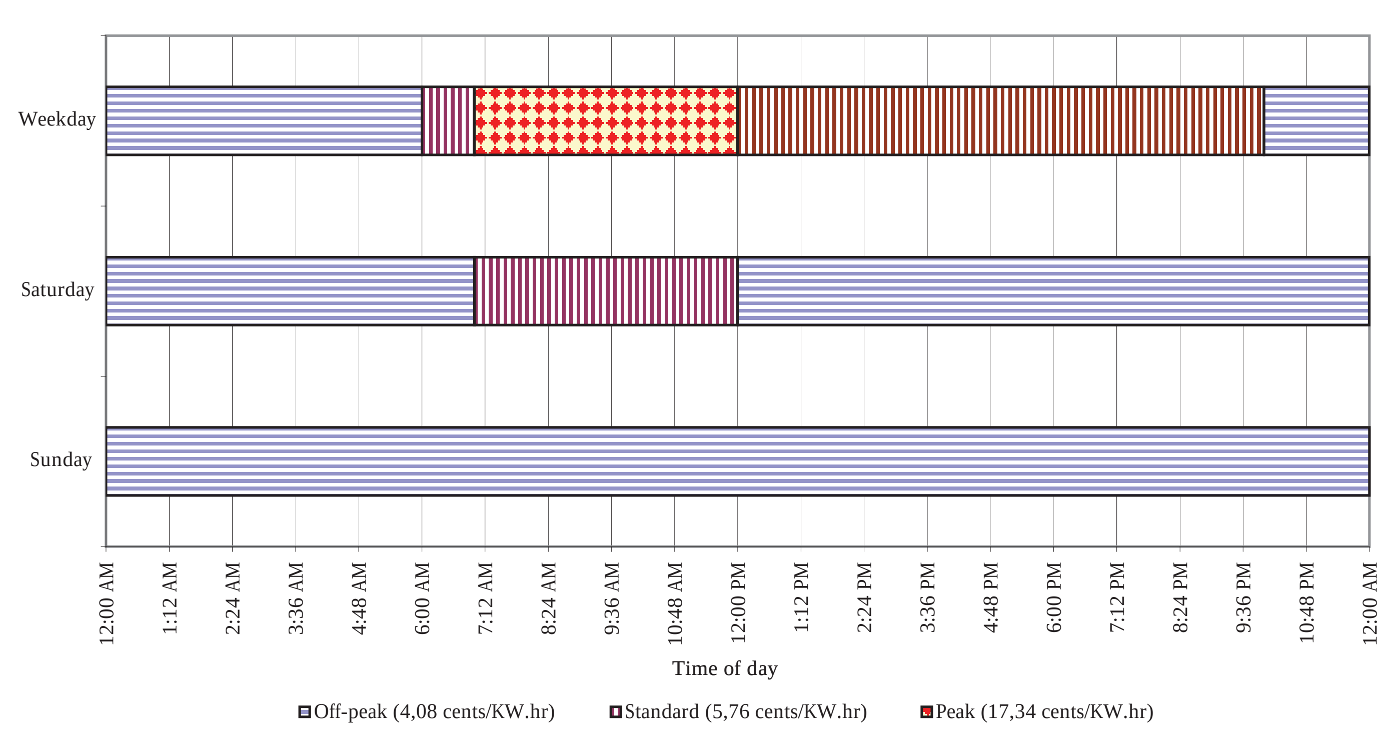


Figure 4 Eskom's active energy tariff

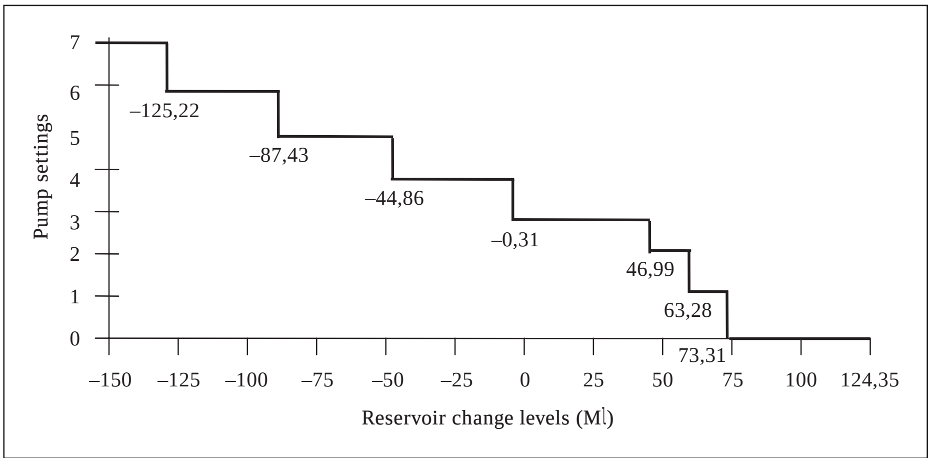


Figure 5 Recommended optimum pumping policy

Table 4 Reservoir limits and penalty costs

Description	Value (Mℓ)	Penalty costs (R/Mℓ)
Maximum	124,35	500,00 (above the Maximum limit)
High	118,14	350,00 (above the High limit)
Low	27,05	500,00 (below the Low limit)
Minimum	00,00	500,00 (below the Minimum limit)

Contrary to the weekly demand sets, the best fit parameter $\phi_{1,1}$ for the monthly demand sets is sufficiently close as are the parameters of the normal distribution used to generate the independent random components.

In summary, the predicted six-month daily demand set is more statistically consistent with its base set than the predicted one-week hourly set is with its base set. The major reason is that the daily demands exhibit a more consistent cyclic pattern than the hourly demands, which, as mentioned earlier, are fictional and chosen haphazardly.

Determining an optimum pumping policy for short-term (hourly) pump operation

The historical four-week and predicted one-week demand sets are shown in figure 3.

From five starting points (random, minimum, even, maximum and centre) shown in table 1 and the demand analysis results (table 2), optimum pumping policies were generated for the four-week demand set (table 5). By running the optimisation from the diverse range of starting points mentioned above, a thorough search for an optimum solution is enhanced. Pump settings and their flow versus reservoir volume equations are shown in table 3. Reservoir penalty, pump switching and storage costs used here are fictional. Reservoir penalty costs are presented in table 4. R25 is charged each time a pump is switched on. –R15 per Mℓ is assigned the storage cost. The negative sign in the storage cost forces the pumping policy to keep reservoir volumes as full as possible over the specified duration while minimising all operating costs. A positive cost will perform the opposite function. Eskom's tariffs are shown in figure 4.

For the four-week period (figure 3), the recommended optimum pumping policy is that generated from starting point 'random' (figure 5). Sufficiently large spaces between each *reservoir change level* allow each pump setting to operate over a reasonable space within the reservoir. In consequence, there are fewer pump switches and consequently, less switching costs than if the *reservoir change levels* were closer to one another. The optimum policy (figure 5) recommends that pump setting 1 (pump 1 only) be operational at any time that the reservoir volume is between 63,28 Mℓ and 73,31 Mℓ and that pump setting 7 (pumps 1, 2 and 3) be operational at any time that the reservoir volume is calculated to be below –125,22 Mℓ. In effect, the optimum pumping policy recommends the singular operation of either pumps 1, 2 or 3 (depending on reservoir volume) as long as the reservoir volume is above the *minimum* (0,00 Mℓ) limit.

The optimum pumping policy (figure 5) is used to simulate the operation of the Libanon system using predicted demands for Monday, 13 May 2004. Figure 6 depicts predicted hourly demands, computed optimum

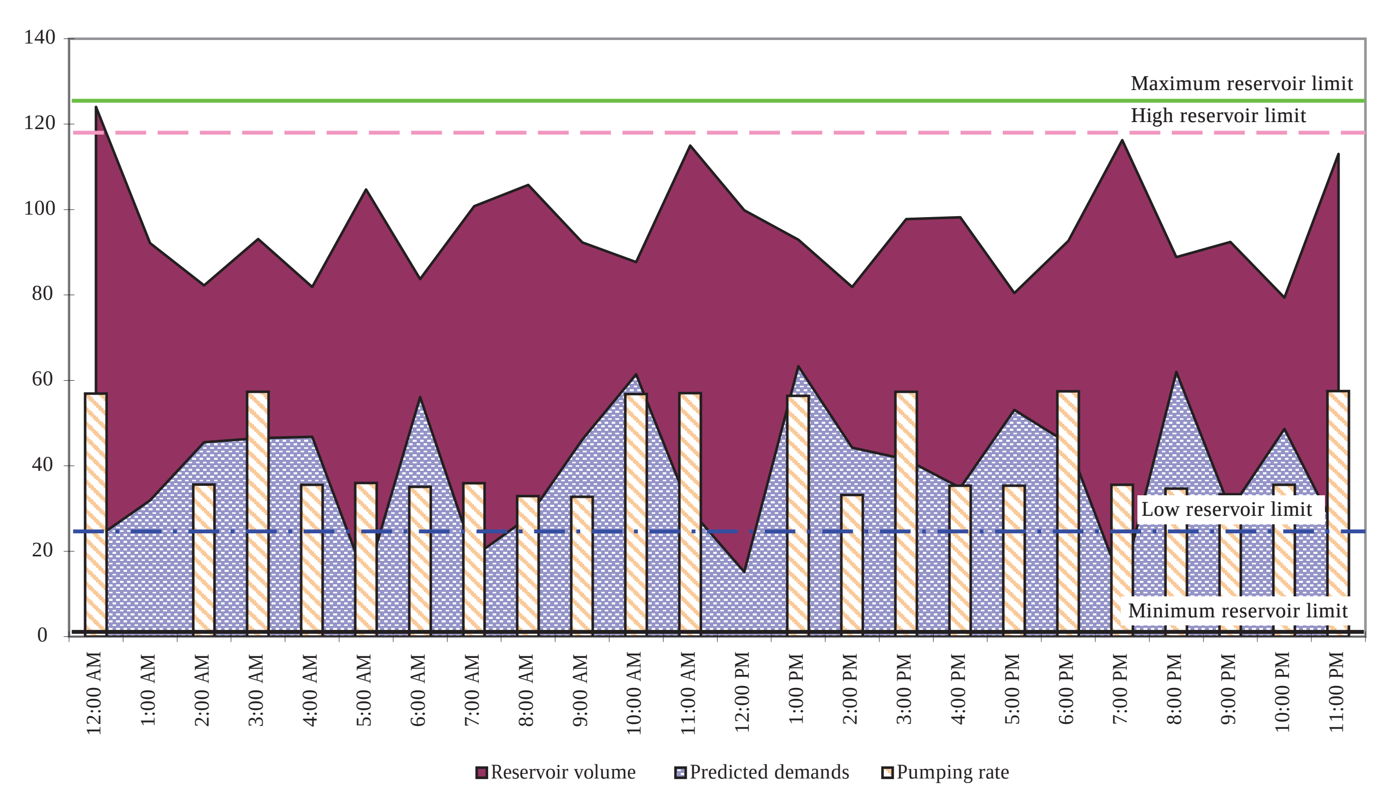


***Figure 6** Predicted hourly demands, optimum pumping rates and reservoir volumes for Monday, 13 May 2004*

***Table 5** Optimum pumping policies and costs for short-term (hourly) operation*

Pump settings	Optimum reservoir change levels (Mℓ) from each starting point				
	Random	Minimum	Even	Maximum	Centre
1	73,3122	69,7482	70,3699	67,8446	78,0136
2	63,2804	66,2586	67,5414	59,3716	60,0570
3	46,9960	49,6820	50,4850	49,1329	47,7981
4	–0,3081	4,5740	4,9919	25,6863	14,3177
5	–44,8551	–21,7444	–42,3929	6,9293	–66,8654
6	–87,4308	–28,7819	–88,3509	–15,1578	–3,4402
7	–125,2155	–49,2485	–128,7236	–41,3472	–81,8449
Reservoir don't change levels	0,00	0,00	0,00	±0,5	0,00
Storage costs (R)	(–) 935 137,88	(–) 947 999,93	(–) 952 797,73	(–) 967 980,02	(–) 960 639,43
Maximum limit penalty costs (R) [number of times the limit was violated]	396,58 [2]	771,75 [1]	1 418,85 [3]	3 059,09 [5]	1 069,54 [2]
High limit penalty costs (R) [number of times the limit was violated]	22 496,29 [24]	21 369,65 [19]	22 824,56 [20]	33 170,29 [31]	23 607,19 [19]
Low limit penalty costs (R)	0,00	0,00	0,00	0,00	0,00
Minimum limit penalty costs (R)	0,00	0,00	0,00	0,00	0,00
Power consumed (KW)	388 403,70	392 120,03	392 016,68	409 564,04	393 844,95
Pumping costs (R)	841 796,76	850 734,34	849 980,36	908 420,54	863 923,83
Pump switching costs (R)	2 725,00	3 450,00	3 450,00	4 850,00	3 050,00
Total costs (R)	1 802 552,51	1 824 325,67	1 830 471,50	1 917 479,94	1 852 289,99

pump flow rates and reservoir volumes over each hour for that day. As can be seen, the volume in the reservoir was kept as full as possible during the entire period by running at the most, one of the three pumps. As long as future demands remain statistically and numerically consistent with historic demands, the pumping policy will remain optimal. It will therefore be unnecessary to recalculate this pumping policy unless there are structural changes in the system or in the statistical properties of the demand. In this case, the most efficient pump (pump 1) is used nearly the whole day, so that the water level does not drop too far towards the *Low* reservoir level.

Determining an optimum pumping policy for long-term (daily) pump operation

Daily demands were predicted over six months – the same period as the historical set (figure 7). Since pumping rates and costs were generated for the historical set, this section aims to validate the DSM presented here. The negative slope in the linear secular trend of both sets is caused by the gradual decrease in demands typically experienced between late spring and late autumn (the seasons over which the data were generated). This slope would likely have been eliminated if at least a full year's worth of historical demands were used in the analysis. To aid comparison, the predicted set is displaced on the ordinate axis by 50 Mℓ (figure 7). Both sets show similar trends and have similar statistical properties (table 2).

The same operating cost components and values are employed here as in the hourly demand example. For the active energy tariffs, Eskom charged the Libanon water supply authority off-peak (18:00–06:00) and standard (06:00–1:00) rates (figure 4) only. The operating costs determined while optimising from each starting point are presented in table 6.

Since it was impossible to ascertain actual storage, penalty and pump switching costs, pumping costs are, in this study, the only appropriate index for comparison between historical and optimised system operation (table 7). Based on pumping alone, the pumping policy from starting point 'Even' generates the least cost of R162 400,27 (table 6). Pumping costs from starting points 'Random', 'Minimum' and 'Centre'

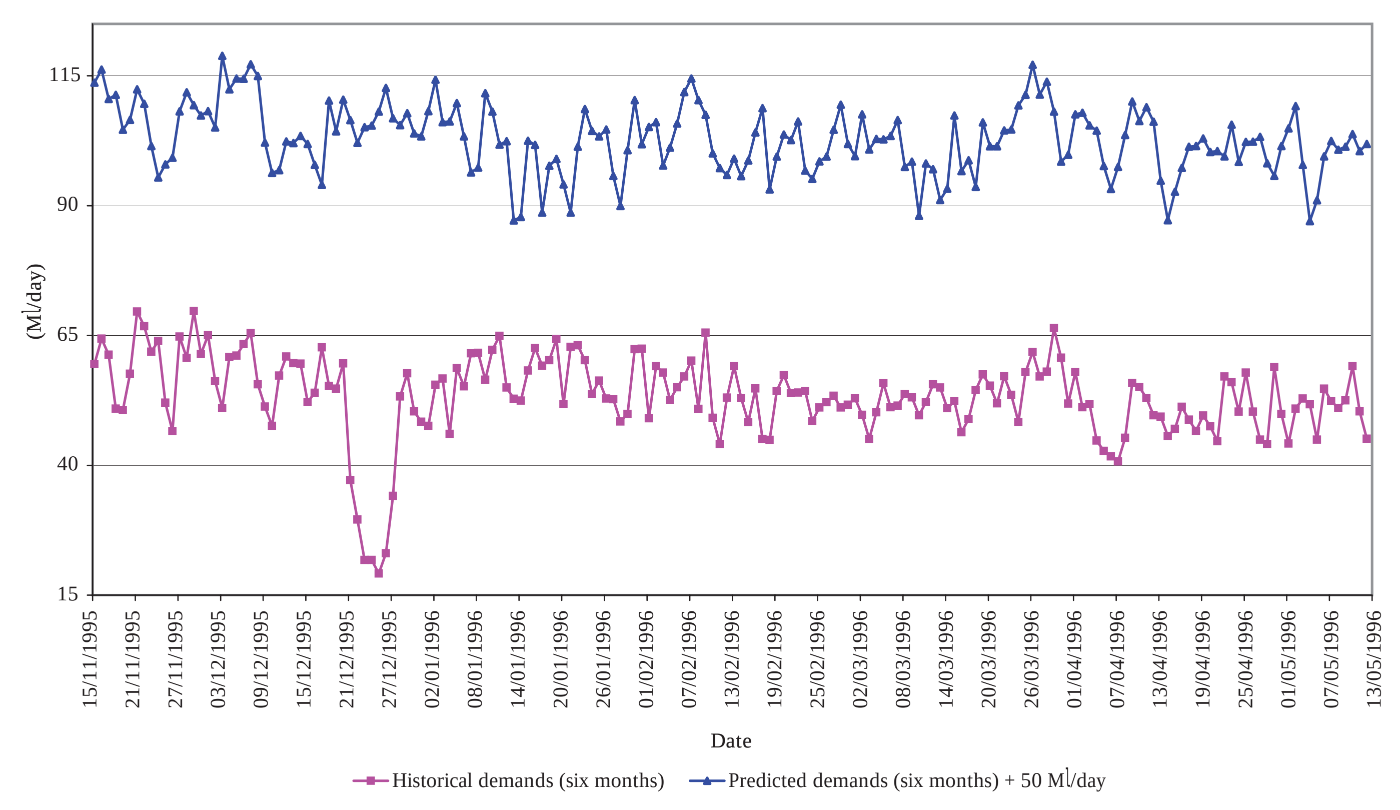


Figure 7 Historical and predicted daily demands

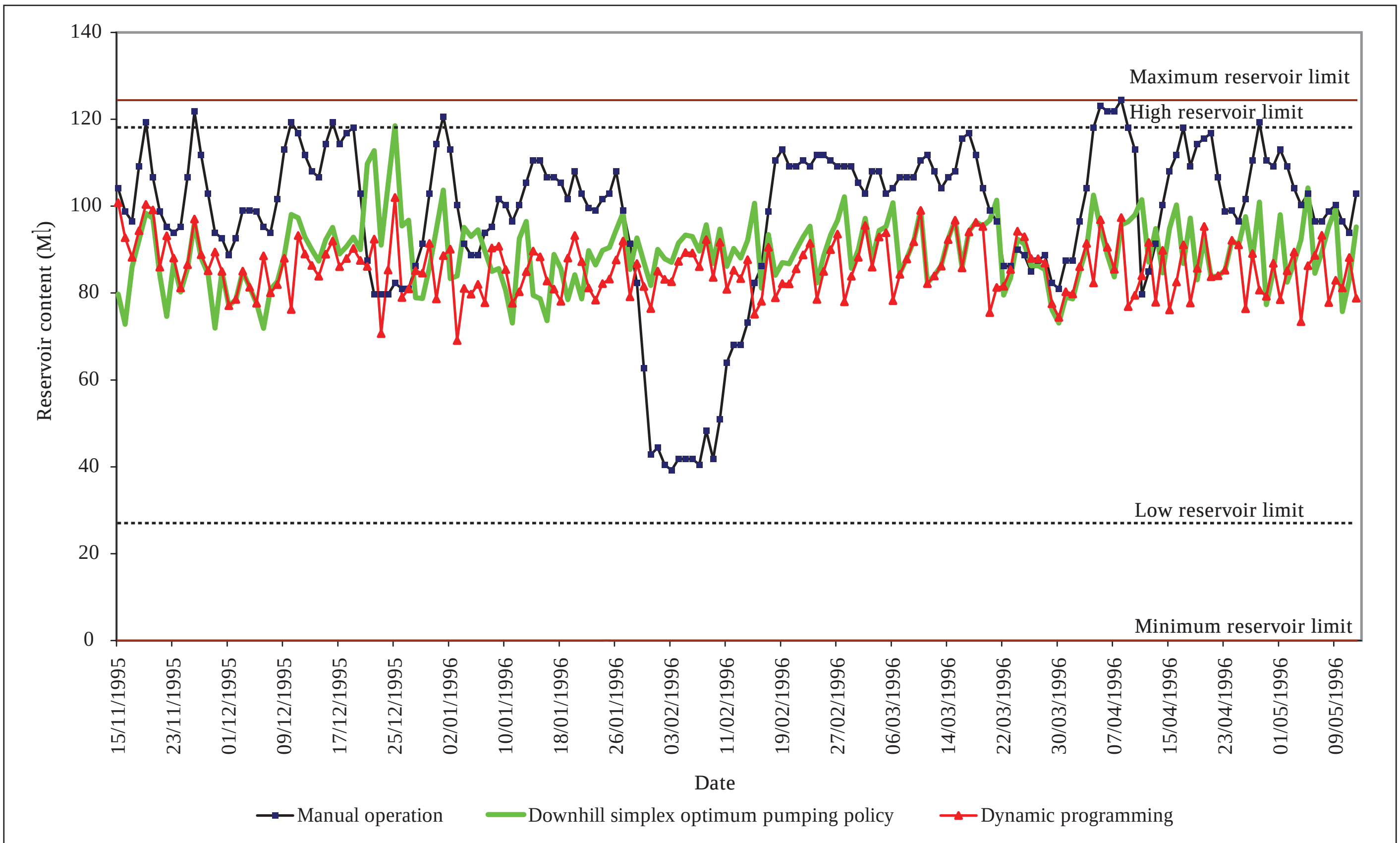


Figure 8 Historical and optimised reservoir volume over six months

are within 1,60 % of this least cost solution, which means that along that cost surface, there is a least cost optimum with small areas of local optima around it. The higher pumping cost of R386 308,69 generated from starting point 'Maximum' may have resulted from either premature stoppage due to the number of function evaluations exceeding the maximum (500 iterations), or the functional standard deviation was within the specified tolerance (0,0001) even though the simplex was still large. The least pumping cost of R162 400,27 is 1,95 % less than that generated using human operators (table 7). Unfortunately, the historical data generated was incapable of describing which pumps were operational at any particular time and penalty costs, if and when applied. The simple DP module confirmed the recommended optimum policy by generating a near global least pumping cost of R161 685,87. This is 2,38 % less than that generated using human operators and 0,44 % less than that calculated using the DSM.

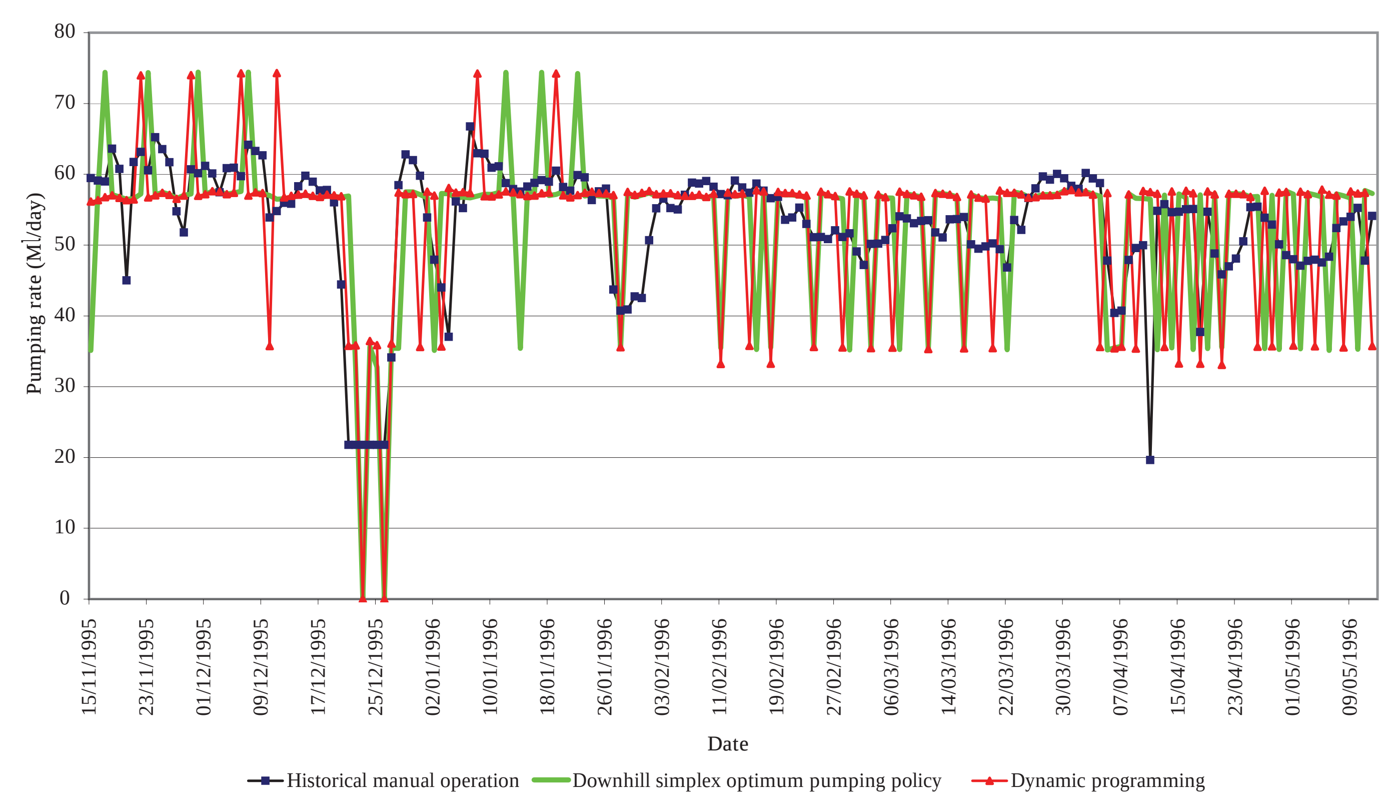


Figure 9 Historical and optimised pumping rates over six months

Table 6 Optimum pumping policy costs for long-term (daily) operation from diverse starting points

Costs	Random	Minimum	Even	Maximum	Centre
Storage costs (R)	(–) 278 076,49	(–) 239 959,96	(–) 241 213,26	(–) 427 046,41	(–) 241 721,07
Maximum limit penalty costs (R) [No of times the limit was violated]	2 234,35 [2]	0,00	0,00	3 043 020,38 [180]	0,00
High limit penalty costs (R) [No of times the limit was violated]	10 737,49 [9]	0,00	128,14 [1]	2 521 848,27 [180]	0,00
Low limit penalty costs (R)	0,00	0,00	0,00	0,00	0,00
Minimum limit penalty costs (R)	0,00	0,00	0,00	0,00	0,00
Power consumed (KW)	128 623,55	126 843,38	126 598,28	301 144,91	127 005,21
Pumping costs (R)	164 998,29	162 714,68	162 400,27	386 308,69	162 922,29
Pump switching costs (R)	225,00	250,00	250,00	75,00	300,00
Total costs (R)	456 271,62	402 924,64	403 991,67	6 378 298,75	404 943,36

Table 7 Comparison between historical and optimised daily pumping costs over six months

Description	Power consumption (KW.hr)	Cost of power for pumping (R)	Cost saving (%)
Historical manual operation	129 113,18	165 626,39	Benchmark
Downhill simplex	126 598,28	162 400,27	+1,95
Dynamic programming	126 041,37	161 685,87	+2,38

Table 8 Effect of demand data length on the determination of an optimum pumping policy

	Demands (60 months)	Extract 1 (30 months)	Extract 2 (15 months)	Extract 3 (71/2 months)
Data points Start date End date	1 800 27/08/1999 31/07/2004	900 20/11/2000 07/05/2003	450 03/07/2001 24/09/2002	225 24/10/2001 04/06/2002
Pump settings	Reservoir change levels (Mℓ)			
1	83,75	86,08	81,37	58,84
2	66,44	68,39	63,78	49,68
3	21,18	24,66	14,12	26,97
4	-56,59	-65,22	-43,13	-43,79
5	-92,66	-96,15	-64,76	-64,63
6	-129,53	-129,36	-88,69	-88,13
7	-164,84	-161,73	-111,89	-110,26
Operating costs (R)	3 312 507,00	3 311 890,00	3 311 340,00	3 312 086,00

The DSM may be seen to be capable of determining pumping policies that are practical and cost effective. Despite the marginal savings in pumping costs in this case study, the DSM presents the added advantage of recommending formal pumping policies that (a) do not require the lengthy training of human operators to achieve, and (b) may be used to successfully operate a manual or semi-automated system.

The higher the storage penalty costs in relation to the other operating cost components, the less likely the pumping policy will violate them. The graphs in figures 8 and 9 clearly show the sensitivity of the DSM and DP to minimising these penalty costs while avoiding violating reservoir limits on a daily basis over the six-month period. The graphs also present a comparison between historical and optimised pump operation.

Effect of data set length on the determination of an optimum pumping policy

Since generating data over long periods of time becomes cumbersome in the absence of a supervisory control and data acquisition system (SCADA), the effect of data set length on the determination of an optimum pumping policy was tested. Sixty months of predicted daily data for the Libanon system were used. From it, three separate data sets, each with differing lengths, were extracted (table 8). An optimum pumping policy was then calculated for each set. Each policy was then used to simulate system operation using the original sixty month demand set. The results indicate a less than 0,035 % difference between the least and other operating costs. This implies that the determination of an optimum pumping policy is negligibly influenced by the length of the demand data used to calculate it.

CONCLUSIONS

The DSM that generates an optimum pumping policy for pump and reservoir system operation is presented here. The DSM determines an optimum pumping policy for short-term as well as long-term system operation by matching different pump settings to reservoir levels for a particular system at the least cost. The simple DP module, which is unable to generate a pumping policy, confirms the DSM pumping policy by optimising the overall pump flow rate on sequential time periods to obtain a near global least cost solution. In tandem, these two methods assist in converging at a near global optimum pumping policy. Operating costs consist of pumping, pump switching, storage and reservoir penalty costs, and these have been incorporated into the pumping policy determination. From the results of the Libanon study presented above, the methodology achieved about 2 % savings in pumping costs alone over a six-month period compared with conventional human intervention operation. Length of base data has been shown to have negligible effect on the generation of an optimum policy and the methodology is significantly sensitive to violating reservoir limits.

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